Branes in the $M_D \times M_{d+} \times M_{d-}$ compactification of type II string on S^1/Z_2 and their cosmological applications

JHEP10(2009)095

(http://iopscience.iop.org/1126-6708/2009/10/095)

Published by IOP Publishing for SISSA
Received: *July 4, 2009*
Revised: *September 15, 2009*
Accepted: *September 29, 2009*
Published: *October 30, 2009*

Branes in the $M_D \times M_{d+} \times M_{d-}$ compactification of type II string on S^1/Z_2 and their cosmological applications

Michael Devin,[a] **Tibra Ali,**[b] **Gerald Cleaver,**[a] **Anzhong Wang**[a] **and Qiang Wu**[a,c]

[a] *CASPER, Department of Physics, Baylor University, Waco, Texas 76798-7316, U.S.A.*
[b] *Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, Ontario, N2L 2Y5, Canada*
[c] *Department of Physics, Zhejiang University of Technology, Hangzhou 310032, China*
E-mail: Michael_Devin@baylor.edu, tali@perimeterinstitute.ca, Gerald_Cleaver@baylor.edu, Anzhong_Wang@baylor.edu, Qiang_Wu@baylor.edu

Abstract: In this paper, we study the implementation of brane worlds in type II string theory. Starting with the NS/NS sector of type II string, we first compactify the $(D + d_+ + d_-)$-dimensional spacetime, and reduce the corresponding action to a D-dimensional effective action, where the topologies and geometries of M_{d_+} and M_{d_-} are arbitrary. We further compactify one of the $(D - 1)$ spatial dimensions on an S^1/Z_2 orbifold, and derive the gravitational and matter field equations both in the bulk and on the branes. Then, we investigate two key issues in such a setup:

(i) the radion stability and radion mass; and

(ii) the localization of gravity, and the corresponding Kaluza-Klein (KK) modes.

We show explicitly that the radion is stable and its mass can be in the order of TeV. In addition, the gravity is localized on the visible brane, and its spectrum of the gravitational KK towers is discrete and can have a mass gap of TeV, too. The high order Yukawa corrections to the 4-dimensional Newtonian potential is exponentially suppressed, and can be negligible. Applying such a setup to cosmology, we obtain explicitly the field equations in the bulk and the generalized Friedmann equations on the branes.

Keywords: Strings and branes phenomenology, Phenomenology of Large extra dimensions

doi:10.1088/1126-6708/2009/10/095

JHEP10(2009)095

Contents

1 Introduction

Brane worlds have been studied extensively in the past decade [1], following Horava and Witten's (HW) ideas [2], where gauge fields of the standard model (SM) are confined on two 9-branes located at the end points of an S^1/Z_2 orbifold. Out of the 9-spatial dimensions of the branes, six are compactified on a very small scale close to the fundamental one. A 5-dimensional effective theory of the 11-dimensional HW heterotic M-Theory on S^1/Z_2 was worked out explicitly by Lukas et al. [3], and shown that the radion is stable [4, 5], and its mass is of the order of 0.1 GeV [5]. In addition, the corresponding tensor perturbations were also studied, and found that the gravity is localized in the visible (TeV) brane [5].

The spectrum of the gravitational Kaluza-Klein (KK) towers is discrete, and the mass gap can be in the order of TeV. The corrections to the 4-dimensional Newtonian potential, due to the high order KK modes, are exponentially suppressed, and are consistent with observations [5]. In such a setup, the long standing *hierarchy problem*, namely the large difference in magnitudes between the Planck and electroweak scales, may be potentially resolved by combining the large extra dimension [6], warped-factor [7] and brane-tension coupling [8] mechanisms. One of the most attractive features of the model, similar to the RS1 model [7], is that it might be soon explored by LHC [9]. For critical reviews of the brane worlds and some open issues, we refer readers to [1].

Another important application of brane worlds is to the cosmological constant problem [10]. In the 4-dimensional spacetimes, there exists Weinberg's no-go theorem for the adjustment of the cosmological constant. However, in higher dimensional spacetimes, the 4-dimensional vacuum energy on the brane does not necessarily give rise to an effective 4-dimensional cosmological constant. Instead, it may only curve the bulk, while leaving the brane still flat [11], whereby Weinberg's no-go theorem is evaded. Along this vein, the cosmological constant problem was studied in the framework of brane worlds in 5-dimensional spacetimes [12] and 6-dimensional supergravity [13]. However, it was soon realized that in the 5-dimensional case hidden fine-tunings are required [14]. In the 6-dimensional case such fine-tunings may not be needed, but it is still not clear whether loop corrections can be as small as expected [15].

In addition, by adding an Einstein-Hilbert term to the brane action, Dvali, Gabadadze and Porrati (DGP) [16] showed that gravity can be altered at immense distances, due to the slow leakage of gravity off our 3-dimensional universe into bulk. It should be noted that the DGP model has only one 3-brane, and the spacetime in the direction perpendicular to the brane is usually infinitely large, in contrast to the RS1 model, where two orbifold branes form the boundary in the transverse direction of the branes, although later Randall and Sundrum proposed another model (RS2), in which only one brane exists [17]. A remarkable feature of the DGP model is that it gives rise to a late cosmic acceleration of the universe, without the introduction of dark energy [18]. It must be noted that, despite of this great success, the DGP model, as well as its hybrids, is usually plagued with the problem of ghost [19, 20], in addition to the problem of the consistency with observations [21, 22].

It should also be noted that the RS1, RS2 and DGP brane worlds, as well as their generalizations [1], are phenomenological models, and how to implement them into string/M theory is still an open question, despite of some important efforts along this direction [23, 24]. Such an implementation turns out to be extremely difficult, as one would expect, given the complexity of the theory. It was exactly because of this that most of the previous works on brane worlds are phenomenological, and should be considered only as an intermediary bridge between observations and fundamental theory.

Lately, as part of the efforts of implementing the RS1 model into string/M theory, the orbifold branes and their applications to cosmology were studied systematically in the framework of both the Horava-Witten heterotic M-Theory [5, 25] and string theory [26–28] on S^1/Z_2. From the point of view of pure numerology, it was found that the 4D effective cosmological constant can be cast in the form,

$$\rho_\Lambda = \frac{\Lambda_4}{8\pi G_4} = 3\left(\frac{R}{l_{pl}}\right)^{\alpha_R}\left(\frac{M}{M_{pl}}\right)^{\alpha_M} M_{pl}^4, \tag{1.1}$$

where R denotes the typical size of the extra dimensions, M the energy scale of string or M theory, and $(\alpha_R, \alpha_M) = (10, 16)$ for string theory [26] and $(\alpha_R, \alpha_M) = (12, 18)$ for the HW heterotic M Theory [25]. In both cases, it can be shown that for $R \simeq 10^{-22}\ m$ and $M \simeq 1\ TeV$, we obtain $\rho_\Lambda \sim \rho_{\Lambda,ob} \simeq 10^{-47}\ GeV^4$. In contrast to that in Einstein's theory, the domination of this term is only temporary. Due to the interaction of the bulk and the brane, the universe will be in its decelerating expansion phase again, whereby all problems connected with a far future de Sitter universe [29, 30] are resolved. This feature was also found in the DGP model [16]. Therefore, a late transient acceleration of the universe seems to be a generic feature of brane worlds.

It was also showed that the radion is stable, and its mass is about 10^{-1} GeV in the Horava-Witten heterotic M-Theory [5] and 10^{-2} GeV in the string theory [28]. Similar to those in M theory, the gravity in string theory is also localized on the visible (TeV) brane. The spectrum of the gravitational KK towers is discrete with a mass gap that can be in the order of TeV. The high order Yukawa corrections to the 4-dimensional effective Newtonian potential are exponentially suppressed.

In this paper, we shall continuously work along the direction of implementing the RS1 model [7] into string/M theory. In particular, In section II, starting with the Neveu-Schwarz/Neveu-Schwarz (NS/NS) sector of type II string, we first consider the compactification of the $(D + d_+ + d_-)$-dimensional spacetime on two manifolds M_{d_+} and M_{d_-}, where the topologies of M_{d_+} and M_{d_-} are unspecified. This opens the possibility of having the dilaton and modulus fields non-zero potentials (masses), which is in contrast to the toroidal compactification considered in [26–28], in which these scalar fields are always massless [31–33]. After reducing the action to an effective D-dimensional one, we further compactify one of the $(D-1)$ spatial dimensions on an S^1/Z_2 orbifold. Lifting it to the original spacetime, they represent $(D + d_+ + d_- - 2)$-dimensional orbiford branes. The corresponding gravitational and matter field equations both in the bulk and on the branes are derived separately in section III, while in section IV such developed formulas are applied to cosmology by setting $D = 5 = d_+ + d_-$. In particular, the generalized Friedmann equations are given explicitly on the branes. In section V the radion stability and radion mass are studied, while in section VI, the tensor perturbations are investigated. It is found that the radion stable, and the gravity is localized on the visible brane. Both the radion mass and the mass gap of the gravitational KK towers can be in the order of TeV, by properly choosing the free parameters presented in the model. The high order Yukawa corrections to the 4-dimensional Newtonian potential, due to the high order KK modes, is exponentially suppressed, and can be negligible. The paper is ended with section VII, in which we summarize our main results and present some remarks to the future work.

To have this paper as much independent as possible, for the sake of reader's convenience, some parts might be repeated from our previous studies of the problems, although we try to limit these to their minimum.

Before proceeding further, we would like to note that, to have a late time accelerating universe from string/M-Theory, Townsend and Wohlfarth [34] invoked a time-dependent compactification of pure gravity in higher dimensions with hyperbolic internal space to circumvent Gibbons' non-go theorem [35]. Their exact solution exhibits a short period of acceleration. The solution is the zero-flux limit of spacelike branes [36]. If non-zero flux or forms are turned on, a transient acceleration exists for both compact internal hyperbolic and flat spaces [37]. Other accelerating solutions by compactifying more complicated time-dependent internal spaces can be found in [38].

We conclude our introduction with a few comments about possible connections that our models may have with supersymmetric theories of braneworld scenarios. Since this is a paper on cosmology, we do not address the issue of supersymmetric embedding or the breaking of supersymmetry. Our approach in this paper is partly string theoretic and partly phenomenological, and hence the supersymmetric embedding of our model deserves further study. Nevertheless, it is worthwhile to flesh out the challenges facing a full supersymmetrization of our model by making some general comments in light of recent important works [39] on supersymmetry in braneworld scenarios.

In [39], the authors have investigated the constraints on braneworlds coming from supersymmetry. In particular, the last paper in [39] subsumes in its analysis several earlier works. In their setup, the authors consider a particularly simple gauged supergravity theory, which is obtained (before the inclusion of branes) by gauging the U(1) subgroup of the symplectic group (SU(2)) of pure $N = 2$ supergravity in five dimensions. (For more general settings in which braneworlds are embedded in gauged supergravities, see [40]). The general conclusion of these works is that to preserve supersymmetry the tensions of the branes have to satisfy a certain inequality. From this point of view the choice of the tensions in the Randall-Sundrum model corresponds to the value of the tensions saturating the bound. However, in these models the cosmological constants on the branes are directly proportional to the tensions. There are solutions to these models which are cosmological (de Sitter to be more precise) and as one would expect from very general grounds these cosmological models break supersymmetry.

We have not investigated what sort of gauged supergravity theories could realize our setup. Although our choice of the brane tensions are identical to that of Randall and Sundrum, we still don't expect supersymmetry to be preserved in our models, due to the fact that we introduce phenomenological Goldberger-Wise fields in the bulk and the branes (however, see [41] for string theoretic sources for such a field). In contrast to the works mentioned above, where the radion is generically set to its vacuum expectation value, we are interested in how a radion potential is induced through the introduction of the Goldberger-Wise field in the presence of some of the background moduli fields. We find a regime in which the radion is stabilized at a value of the cosmological constant that is positive. One then expects that a necessary condition for supersymmetry would be when the minimum $V_\Phi(Y_c^{\min})$ of the potential is either zero or negative. But we find that in the limit when $V_\Phi(Y_c^{\min}) \to 0$, the two branes coalesce (i.e., $Y_c^{\min} \to 0$). Since we expect quantum gravity corrections to be strong in that limit, we cannot trust the supergravity lagrangian that we started from. Thus in a sense we are restricted to the cosmological regime in our paper, and it will require further calculations to make more precise connection with gauge supergravity braneworld models. We leave this as an exercise for the future. The importance of this exercise stems from the fact that the precise mechanism of supersymmetry breaking will control the higher order corrections to the potential explored here.

2 The model

In this section, we consider the compactification of the NS/NS sector in $(D + d_+ + d_-)$-dimensions, and obtain an effective D-dimensional action. Then, we compactify one of the $(D-1)$ spatial dimensions by introducing two orbifold branes as the boundaries along this compactified dimension.

2.1 Compactification of the NS/NS sector

Let us consider the NS/NS sector in $(D+d_+ +d_-)$-dimensions, $\hat{M}_N = M_D \times \mathcal{M}_{d_+} \times \mathcal{M}_{d_-}$, where $\mathcal{M}_{d_+}$ and $\mathcal{M}_{d_-}$ are d_+ and d_- dimensional spaces, respectively, and $N \equiv D+d_+ +d_-$. To have our formulas as much applicable as possible, we shall not specify the topologies of these spaces. The action takes the form [31, 32, 42],

$$\hat{S}_N = -\frac{1}{2\kappa_N^2}\int d^N x\sqrt{|\hat{g}_N|}e^{-\hat{\Phi}} \times \left\{\hat{R}_N[\hat{g}] + \left(\hat{\nabla}\hat{\Phi}\right)^2 - \frac{1}{12}\hat{H}^2\right\}, \tag{2.1}$$

where $\hat{\nabla}$ denotes the covariant derivative with respect to $\hat{g}^{AB}$ with $A, B = 0, 1, \ldots, N-1$, and $\hat{\Phi}$ is the dilaton field. The NS three-form field $\hat{H}_{ABC}$ is defined as

$$\hat{H}_{ABC} = 3\partial_{[A}\hat{B}_{BC]} = \partial_A\hat{B}_{BC} + \partial_B\hat{B}_{CA} + \partial_C\hat{B}_{AB}, \tag{2.2}$$

where the square brackets imply total antisymmetrization over all indices, and

$$\hat{B}_{CD} = -\hat{B}_{DC}, \quad \partial_A\hat{B}_{CD} \equiv \frac{\partial\hat{B}_{CD}}{\partial x^A}. \tag{2.3}$$

The constant κ_N^2 denotes the gravitational coupling constant, defined as

$$\kappa_N^2 = 8\pi G_N = \frac{1}{M_N^{N-2}}, \tag{2.4}$$

where G_N and M_N denote, respectively, the N-dimensional Newtonian constant and Planck mass.

In this paper we consider the N-dimensional spacetimes described by the metric,

$$\begin{aligned} d\hat{s}_N^2 &= \hat{g}_{AB}dx^A dx^B \\ &= \tilde{g}_{ab}\left(x\right)dx^a dx^b + e^{\sqrt{\frac{2}{d_+}}\psi_+(x)}h^+_{ij}\left(z_+\right)dz_+^i dz_+^j + e^{\sqrt{\frac{2}{d_-}}\psi_-(x)}h^-_{pq}\left(z_-\right)dz_-^p dz_-^q, \end{aligned} \tag{2.5}$$

where $\tilde{g}_{ab}(x)$ is the metric on M_D, parametrized by the coordinates x^a with $a, b, c = 0, 1, \ldots, D-1$, $h^+_{ij}\left(z_+\right)$ the metric on the compact space $\mathcal{M}_{d_+}$ with coordinates z_+^i, where $i, j = D, D+1, \ldots, D+d_+ -1$, and $h^-_{ij}\left(z_-\right)$ the metric on the compact space $\mathcal{M}_{d_-}$ with coordinates z_-^p, where $p, q = D+d_+, D+d_+ +1, \ldots, N-1$.

JHEP10(2009)095

We assume that the daliton field $\hat{\Phi}$ is function of x^a, and the flux $\hat{B}_{CD}$ is block diagonal,

$$\left(\hat{B}_{CD}\right) = \begin{pmatrix} B_{ab}(x) & 0 & 0 \\ 0 & e^{\xi_+(x)}B_{ij}\left(z_+\right) & 0 \\ 0 & 0 & e^{\xi_-(x)}B_{pq}\left(z_-\right) \end{pmatrix}. \tag{2.6}$$

Then, it can be shown that the non-vanishing components of $\hat{H}_{ABC}$ are

$$\begin{aligned} \hat{H}_{abc} &= H_{abc} = 3\partial_{[a}B_{bc]}, \\ \hat{H}_{ijk} &= e^{\xi_+}H_{ijk} = 3e^{\xi_+}\partial_{[i}B_{jk]}, \\ \hat{H}_{pqr} &= e^{\xi_-}H_{pqr} = 3e^{\xi_-}\partial_{[p}B_{qr]}, \\ \hat{H}_{aij} &= B_{ij}e^{\xi_+}\tilde{\nabla}_a\xi_+, \\ \hat{H}_{apq} &= B_{pq}e^{\xi_-}\tilde{\nabla}_a\xi_-, \end{aligned} \tag{2.7}$$

where $\tilde{\nabla}_a$ denotes the covariant derivative with respect to $\tilde{g}^{ab}$. On the other hand, we also have

$$\begin{aligned} \hat{R}_N[\hat{g}] &= \tilde{R}_D[\tilde{g}] + e^{-\sqrt{\frac{2}{d_+}}\psi_+}R_{d_+}\left[h^+\right] + e^{-\sqrt{\frac{2}{d_-}}\psi_-}R_{d_-}\left[h^-\right] - 2\tilde{g}^{ab}\tilde{\nabla}_a\tilde{\nabla}_bQ \\ &\quad -\frac{(d_+ + 1)}{2}\left(\tilde{\nabla}\psi_+\right)^2 - \frac{(d_- + 1)}{2}\left(\tilde{\nabla}\psi_-\right)^2 - \sqrt{d_+d_-}\left(\tilde{\nabla}\psi_+\right)\left(\tilde{\nabla}\psi_-\right), \end{aligned} \tag{2.8}$$

where

$$Q \equiv \sqrt{\frac{d_+}{2}}\,\psi_+ + \sqrt{\frac{d_-}{2}}\,\psi_-. \tag{2.9}$$

Making the following conformal transformations,

$$g_{ab} = \Omega^2\tilde{g}_{ab}, \quad \Omega = e^{\frac{Q-\hat{\Phi}}{D-2}}, \tag{2.10}$$

we find that

$$\begin{aligned} \tilde{R}_D[\tilde{g}] &= \Omega^2\left\{R_D[g] + 2(D-1)\Box\ln\Omega - (D-2)(D-1)\left(\nabla\ln\Omega\right)^2\right\}, \\ \tilde{g}^{ab}\tilde{\nabla}_a\tilde{\nabla}_bQ &= \Omega^2\Big(\Box Q - (D-2)\left(\nabla Q\right)\left(\nabla\ln\Omega\right)\Big), \end{aligned} \tag{2.11}$$

where $\Box \equiv g^{ab}\nabla_a\nabla_b$, and ∇_a denotes the covariant derivative with respect to g^{ab}. Then, combining eqs. (2.8) and (2.11), we obtain

$$\begin{aligned} &\sqrt{|\hat{g}_N|}e^{-\hat{\Phi}}\left\{\hat{R}_N[\hat{g}] + \left(\hat{\nabla}\hat{\Phi}\right)^2 - \frac{1}{12}\hat{H}^2\right\} \\ &= \sqrt{|g_Dh^+h^-|}\Bigg\{R_D[g] + e^{-2\frac{Q-\hat{\Phi}}{D-2}}\left(e^{-\sqrt{\frac{2}{d_+}}\psi_+}R_{d_+} + e^{-\sqrt{\frac{2}{d_-}}\psi_-}R_{d_-} - \frac{1}{12}\hat{H}^2\right) + \frac{2}{D-2}\Box Q \\ &\qquad -\frac{2(D-1)}{D-2}\Box\hat{\Phi} - \frac{1}{D-2}\left(\nabla\left(Q-\hat{\Phi}\right)\right)^2 - \frac{1}{2}\left(\nabla\psi_+\right)^2 - \frac{1}{2}\left(\nabla\psi_-\right)^2\Bigg\}, \end{aligned} \tag{2.12}$$

where

$$\hat{H}^2 = e^{\frac{6(Q-\hat{\Phi})}{D-2}} H^2 + 3e^{\frac{2(Q-\hat{\Phi})}{D-2}} \left(e^{2\left(\xi_+ - \sqrt{\frac{2}{d_+}}\psi_+\right)} B_+^2 \left(\nabla \xi_+\right)^2 + e^{2\left(\xi_- - \sqrt{\frac{2}{d_-}}\psi_-\right)} B_-^2 \left(\nabla \xi_-\right)^2 \right)$$
$$+ e^{2\xi_+ - 3\sqrt{\frac{2}{d_+}}\psi_+} H_+^2 + e^{2\xi_- - 3\sqrt{\frac{2}{d_-}}\psi_-} H_-^2, \qquad (2.13)$$

with

$$\begin{aligned} H^2 &= H_{abc}(x) H^{abc}(x), \\ H_+^2 &= H_{ijk}\left(z_+\right) H^{ijk}\left(z_+\right), \\ H_-^2 &= H_{pqr}\left(z_-\right) H^{pqr}\left(z_-\right), \\ B_+^2 &= B_{ij}\left(z_+\right) B^{ij}\left(z_+\right), \\ B_-^2 &= B_{pq}\left(z_-\right) B^{pq}\left(z_-\right), \end{aligned} \qquad (2.14)$$

and

$$g^{ab} g_{ac} = \delta_c^b, \quad h^{+ik} h_{ij}^+ = \delta_j^k, \quad h^{-pq} h_{pr}^- = \delta_r^q. \qquad (2.15)$$

Substituting eqs. (2.13) and (2.14) into eq. (2.1), and then integrating it by part, we obtain the $D-$dimensional effective action in the Einstein frame,

$$S_D^{(E)} = -\frac{1}{2\kappa_D^2} \int \sqrt{|g_D|} d^D x \left(R_D[g] - \mathcal{L}_D^{(E)}\left(\phi_n, \xi_\pm\right) \right), \qquad (2.16)$$

where $\phi_n = \{\phi, \psi_\pm\}$, and

$$\kappa_D^2 \equiv \frac{\kappa_N^2}{V_{d_+} V_{d_-}}, \qquad (2.17)$$

$$V_{d_\pm} \equiv \int \sqrt{|h^\pm|}\, d^{d_\pm} z_\pm,$$
$$\mathcal{L}_D^{(E)} = \frac{1}{2} \sum_n \left(\nabla \phi_n\right)^2 + \frac{1}{12} e^{-\sqrt{\frac{8}{D-2}}\phi} H^2 + \alpha_+ e^{2\xi_+ - \sqrt{\frac{8}{d_+}}\psi_+} \left(\nabla \xi_+\right)^2 + \alpha_- e^{2\xi_- - \sqrt{\frac{8}{d_-}}\psi_-} \left(\nabla \xi_-\right)^2$$
$$- e^{\sqrt{\frac{2}{D-2}}\phi} \left(\beta_+ e^{-\sqrt{\frac{2}{d_+}}\psi_+} + \beta_- e^{-\sqrt{\frac{2}{d_-}}\psi_-} - \gamma_+ e^{2\xi_+ - \sqrt{\frac{18}{d_+}}\psi_+} - \gamma_- e^{2\xi_- - \sqrt{\frac{18}{d_-}}\psi_-} \right), \qquad (2.18)$$

and

$$\phi \equiv \sqrt{\frac{2}{D-2}} \left(\hat{\Phi} - Q \right), \qquad (2.19)$$

$$\begin{aligned} \alpha_\pm &\equiv \frac{1}{4V_{d_\pm}} \int d^{d_\pm} z_\pm \sqrt{|h^\pm|} B_\pm^2\left(z_\pm\right), \\ \beta_\pm &\equiv \frac{1}{V_{d_\pm}} \int d^{d_\pm} z_\pm \sqrt{|h^\pm|} R_{d_\pm}\left(z_\pm\right), \\ \gamma_\pm &\equiv \frac{1}{12 V_{d_\pm}} \int d^{d_\pm} z_\pm \sqrt{|h^\pm|} H_\pm^2\left(z_\pm\right). \end{aligned} \qquad (2.20)$$

JHEP10(2009)095

2.2 S^1/Z_2 compactification of the D-dimensional sector

We shall compactify one of the $(D-1)$ spatial dimensions by placing two orbifold branes as its boundaries. The brane actions are taken as,

$$S^{(E,I)}_{D-1,m} = -\epsilon_I \int_{M^{(I)}_{D-1}} \sqrt{\left|g^{(I)}_{D-1}\right|} V^{(I)}_{D-1}\left(\phi_n, \xi_\pm\right) d^{D-1}\xi_{(I)} + \int_{M^{(I)}_{D-1}} d^{D-1}\xi_{(I)} \sqrt{\left|g^{(I)}_{D-1}\right|} \times \mathcal{L}^{(I)}_{D-1,m}\left(\phi_n, \xi_\pm, \chi\right), \tag{2.21}$$

where $I = 1, 2$, $V^{(I)}_{D-1}(\phi_n, \xi_\pm)$ denotes the potential of the scalar fields ϕ_n on the branes, and $\xi^{\mu}_{(I)}$'s are the intrinsic coordinates of the branes with $\mu, \nu = 0, 1, 2, \ldots, D-2$, and $\epsilon_1 = -\epsilon_2 = 1$. χ denotes collectively the matter fields. The two branes are localized on the surfaces,

$$\Phi_I\left(x^a\right) = 0, \tag{2.22}$$

or equivalently

$$x^a = x^a\left(\xi^{\mu}_{(I)}\right). \tag{2.23}$$

$g^{(I)}_{D-1}$ denotes the determinant of the reduced metric $g^{(I)}_{\mu\nu}$ of the I-th brane, defined as

$$g^{(I)}_{\mu\nu} \equiv g_{ab} e^{(I)a}_{(\mu)} e^{(I)b}_{(\nu)}\Big|_{M^{(I)}_{D-1}}, \tag{2.24}$$

where

$$e^{(I)\,a}_{(\mu)} \equiv \frac{\partial x^a}{\partial \xi^{\mu}_{(I)}}. \tag{2.25}$$

Then, the total action is given by,

$$S^{(E)}_{\text{total}} = S^{(E)}_D + \sum_{I=1}^{2} S^{(E,I)}_{D-1,m}. \tag{2.26}$$

3 Field equations both outside and on the orbifold branes

Variation of the total action (2.26) with respect to the metric g_{ab} yields the field equations,

$$G^{(D)}_{ab} = \kappa^2_D T^{(D)}_{ab} + \kappa^2_D \sum_{I=1}^{2} \mathcal{T}^{(I)}_{\mu\nu} e^{(I,\,\mu)}_a e^{(I,\,\nu)}_b \times \sqrt{\left|\frac{g^{(I)}_{D-1}}{g_D}\right|}\, \delta\left(\Phi_I\right), \tag{3.1}$$

where $\delta(x)$ denotes the Dirac delta function, normalized in the sense of [43], and the energy-momentum tensors $T^{(D)}_{ab}$ and $\mathcal{T}^{(I)}_{\mu\nu}$ are defined as,

$$\kappa^2_D T^{(D)}_{ab} \equiv \frac{1}{2}\left(\nabla_a \phi^n\right)\left(\nabla_b \phi_n\right) + \alpha_+ e^{2\xi_+ - \sqrt{\frac{8}{d_+}}\psi_+}\left(\nabla_a \xi_+\right)\left(\nabla_b \xi_+\right) + \alpha_- e^{2\xi_- - \sqrt{\frac{8}{d_-}}\psi_-}\left(\nabla_a \xi_-\right)\left(\nabla_b \xi_-\right) + \frac{1}{4} e^{-\sqrt{\frac{8}{D-2}}\,\phi} H_{acd} H_b{}^{cd} - \frac{1}{2} g_{ab} \mathcal{L}^{(E)}_D, \tag{3.2}$$

$$\mathcal{T}^{(I)}_{\mu\nu} \equiv \mathcal{S}^{(I)}_{\mu\nu} + \tau^{(I)}_p g^{(I)}_{\mu\nu},$$

$$\mathcal{S}^{(I)}_{\mu\nu} \equiv 2\frac{\delta \mathcal{L}^{(I)}_{D-1,m}}{\delta g^{(I)\,\mu\nu}} - g^{(I)}_{\mu\nu} \mathcal{L}^{(I)}_{D-1,m}, \tag{3.3}$$

where $\phi^n = \phi_n$, and

$$\tau_p^{(I)} \equiv \epsilon_I V_{D-1}^{(I)} \left(\phi_n, \xi_\pm\right). \tag{3.4}$$

Variation of the total action (2.26), respectively, with respect to ϕ, $\psi_\pm$, $\xi_\pm$ and B_{ab}, yields the following equations of the matter fields,

$$\begin{aligned}
\Box\phi &= -\frac{1}{12}\sqrt{\frac{8}{D-2}}e^{-\sqrt{\frac{8}{D-2}}\,\phi}H^2 \\
&\quad -\sqrt{\frac{2}{D-2}}e^{\sqrt{\frac{8}{D-2}}\,\phi}\left(\beta_+ e^{-\sqrt{\frac{2}{d_+}}\,\psi_+} + \beta_- e^{-\sqrt{\frac{2}{d_-}}\,\psi_-} - \gamma_+ e^{2\xi_+ - \sqrt{\frac{18}{d_+}}\,\psi_+} - \gamma_- e^{2\xi_- - \sqrt{\frac{18}{d_-}}\,\psi_-}\right) \\
&\quad -\sum_{i=1}^{2}\left(2\kappa_D^2\epsilon_I\frac{\partial V_{D-1}^{(I)}}{\partial\phi} + \sigma_\phi^{(I)}\right)\times\sqrt{\left|\frac{g_{D-1}^{(I)}}{g_D}\right|}\,\delta\left(\Phi_I\right),
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
\Box\psi_\pm &= -\alpha_\pm\sqrt{\frac{8}{d_\pm}}e^{2\xi_\pm - \sqrt{\frac{8}{d_\pm}}\psi_\pm}\left(\nabla\xi_\pm\right)^2 e^{\sqrt{\frac{2}{D-2}}\,\phi}\left(\beta_\pm\sqrt{\frac{2}{d_\pm}}e^{-\sqrt{\frac{2}{d_\pm}}\psi_\pm} - \gamma_\pm\sqrt{\frac{18}{d_\pm}}e^{2\xi_\pm - \sqrt{\frac{18}{d_\pm}}\psi_\pm}\right) \\
&\quad -\sum_{i=1}^{2}\left(2\kappa_D^2\epsilon_I\frac{\partial V_{D-1}^{(I)}}{\partial\psi_\pm} + \sigma_{\psi_\pm}^{(I)}\right)\times\sqrt{\left|\frac{g_{D-1}^{(I)}}{g_D}\right|}\,\delta\left(\Phi_I\right),
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
\Box\xi_\pm &= -\left(\nabla\xi_\pm\right)^2 + \sqrt{\frac{8}{d_\pm}}\left(\nabla_a\xi_\pm\right)\left(\nabla^a\psi_\pm\right) + \frac{\gamma_\pm}{\alpha_\pm}e^{\sqrt{\frac{2}{D-2}}\,\phi - \sqrt{\frac{2}{d_\pm}}\psi_\pm} - \frac{1}{2\alpha_\pm}e^{\sqrt{\frac{8}{d_\pm}}\psi_\pm - 2\xi_\pm} \\
&\quad \times\sum_{I=1}^{2}\left(2\kappa_D^2\epsilon_I\frac{\partial V_{D-1}^{(I)}}{\partial\xi_\pm} + \sigma_{\xi_\pm}^{(I)}\right)\times\sqrt{\left|\frac{g_{D-1}^{(I)}}{g_D}\right|}\,\delta\left(\Phi_I\right),
\end{aligned} \tag{3.7}$$

$$\nabla^c H_{cab} = \sqrt{\frac{8}{D-2}}\,H_{cab}\nabla^c\phi - \sum_{i=1}^{2}\sigma_{ab}^{(I)}\sqrt{\left|\frac{g_{D-1}^{(I)}}{g_D}\right|}\,\delta\left(\Phi_I\right), \tag{3.8}$$

where

$$\begin{aligned}
\sigma_\phi^{(I)} &\equiv -2\kappa_D^2\frac{\delta\mathcal{L}_{D-1,m}^{(I)}}{\delta\phi}, \\
\sigma_{\psi_\pm}^{(I)} &\equiv -2\kappa_D^2\frac{\delta\mathcal{L}_{D-1,m}^{(I)}}{\delta\psi_\pm}, \\
\sigma_{\xi_\pm}^{(I)} &\equiv -2\kappa_D^2\frac{\delta\mathcal{L}_{D-1,m}^{(I)}}{\delta\xi_\pm}, \\
\sigma_{ab}^{(I)} &\equiv -4\kappa_D^2 e^{\sqrt{\frac{8}{D-2}}\,\phi}\frac{\delta\mathcal{L}_{D-1,m}^{(I)}}{\delta B^{ab}}.
\end{aligned} \tag{3.9}$$

Eq. (3.1) and eqs. (3.5)–(3.8) consist of the complete set of the gravitational and matter field equations. To solve these equations, it is found very convenient to separate them into two groups, one is defined outside the two orbifold branes, and the other is defined on the two branes.

3.1 Field equations outside the two branes

To write down the equations outside the two orbifold branes is straightforward, and they are simply the D-dimensional gravitational field equations (3.1), and the matter field equations eqs. (3.5)–(3.8) without the delta function parts,

$$\begin{aligned}\Box\phi &= -\frac{1}{12}\sqrt{\frac{8}{D-2}}e^{-\sqrt{\frac{8}{D-2}}\,\phi}H^2 - \sqrt{\frac{2}{D-2}}e^{\sqrt{\frac{8}{D-2}}\,\phi} \\ &\quad\times\left(\beta_+ e^{-\sqrt{\frac{2}{d_+}}\,\psi_+} + \beta_- e^{-\sqrt{\frac{2}{d_-}}\,\psi_-} - \gamma_+ e^{2\xi_+ - \sqrt{\frac{18}{d_+}}\,\psi_+} - \gamma_- e^{2\xi_- - \sqrt{\frac{18}{d_-}}\,\psi_-}\right), \qquad (3.10)\end{aligned}$$

$$\begin{aligned}\Box\psi_\pm &= -\alpha_\pm\sqrt{\frac{8}{d_\pm}}e^{2\xi_\pm - \sqrt{\frac{8}{d_\pm}}\psi_\pm}\left(\nabla\xi_\pm\right)^2 e^{\sqrt{\frac{2}{D-2}}\,\phi} \\ &\quad\times\left(\beta_\pm\sqrt{\frac{2}{d_\pm}}e^{-\sqrt{\frac{2}{d_\pm}}\psi_\pm} - \gamma_\pm\sqrt{\frac{18}{d_\pm}}e^{2\xi_\pm - \sqrt{\frac{18}{d_\pm}}\psi_\pm}\right), \qquad (3.11)\end{aligned}$$

$$\Box\xi_\pm = -\left(\nabla\xi_\pm\right)^2 + \sqrt{\frac{8}{d_\pm}}\left(\nabla_a\xi_\pm\right)\left(\nabla^a\psi_\pm\right) + \frac{\gamma_\pm}{\alpha_\pm}e^{\sqrt{\frac{2}{D-2}}\,\phi - \sqrt{\frac{2}{d_\pm}}\psi_\pm}, \qquad (3.12)$$

$$\nabla^c H_{cab} = \sqrt{\frac{8}{D-2}}\,H_{cab}\nabla^c\phi. \qquad (3.13)$$

Therefore, in the rest of this section, we shall concentrate ourselves on the derivation of the field equations on the branes.

3.2 Field equations on the two branes

To write down the field equations on the two orbifold branes, one can follow two different approaches: (i) First express the delta function parts in the left-hand sides of eqs. (3.1) and (3.5)–(3.8) in terms of the discontinuities of the first derivatives of the metric coefficients and matter fields, and then equal the corresponding delta function parts in the right-hand sides of these equations, as shown systematically in [44, 45]. (ii) The second approach is to use the Gauss-Codacci and Lanczos equations to write down the $(D-1)$-dimensional gravitational field equations on the branes [46]. It should be noted that these two approaches are equivalent and complementary one to the other. In this paper, we shall follow the second approach to write down the gravitational field equations on the two branes, and the first approach to write the matter field equations on the two branes.

3.2.1 Gravitational field equations on the two branes

From the Gauss-Codacci equations, we we obtain [46],

$$G^{(D-1)}_{\mu\nu} = \mathcal{G}^{(D)}_{\mu\nu} + E^{(D)}_{\mu\nu} + \mathcal{F}^{(D-1)}_{\mu\nu}, \qquad (3.14)$$

with

$$\begin{aligned}\mathcal{G}^{(D)}_{\mu\nu} &\equiv \frac{D-3}{(D-2)}\left\{G^{(D)}_{ab}e^{a}_{(\mu)}e^{b}_{(\nu)} - \left[G_{ab}n^a n^b + \frac{1}{D-1}G^{(D)}\right]g_{\mu\nu}\right\}, \\ E^{(D)}_{\mu\nu} &\equiv C^{(D)}_{abcd}n^a e^{b}_{(\mu)}n^c e^{d}_{(\nu)}, \\ \mathcal{F}^{(D-1)}_{\mu\nu} &\equiv K_{\mu\lambda}K^{\lambda}_{\nu} - KK_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\left(K_{\alpha\beta}K^{\alpha\beta} - K^2\right), \qquad (3.15)\end{aligned}$$

where n^a denotes the normal vector to the brane, $G^{(D)} \equiv g^{ab}G^{(D)}_{ab}$, and $C^{(D)}_{abcd}$ the Weyl tensor. The extrinsic curvature $K_{\mu\nu}$ is defined as

$$K_{\mu\nu} \equiv e^a_{(\mu)} e^b_{(\nu)} \nabla_a n_b. \tag{3.16}$$

A crucial step of this approach is the Lanczos equations [47],

$$\left[K^{(I)}_{\mu\nu}\right]^- - g^{(I)}_{\mu\nu}\left[K^{(I)}\right]^- = -\kappa^2_D \mathcal{T}^{(I)}_{\mu\nu}, \tag{3.17}$$

where

$$\begin{aligned}\left[K^{(I)}_{\mu\nu}\right]^- &\equiv \lim_{\Phi_I \to 0^+} K^{(I)\,+}_{\mu\nu} - \lim_{\Phi_I \to 0^-} K^{(I)\,-}_{\mu\nu}, \\ \left[K^{(I)}\right]^- &\equiv g^{(I)\,\mu\nu}\left[K^{(I)}_{\mu\nu}\right]^-. \end{aligned} \tag{3.18}$$

Assuming that the branes have Z_2 symmetry, we can express the intrinsic curvatures $K^{(I)}_{\mu\nu}$ in terms of the effective energy-momentum tensor $\mathcal{T}^{(I)}_{\mu\nu}$ through the Lanczos equations (3.17). Setting

$$\mathcal{S}^{(I)}_{\mu\nu} = \tau^{(I)}_{\mu\nu} + g^{(I)}_k g^{(I)}_{\mu\nu}, \tag{3.19}$$

where $g^{(I)}_k$ is a coupling constant of the I-th brane [8], we find that

$$\mathcal{T}^{(I)}_{\mu\nu} = \tau^{(I)}_{\mu\nu} + \left(g^{(I)}_k + \tau^{(I)}_p\right) g^{(I)}_{\mu\nu}. \tag{3.20}$$

Then, $G^{(D-1)}_{\mu\nu}$ given by eq. (3.14) can be cast in the form,

$$G^{(D-1)}_{\mu\nu} = \mathcal{G}^{(D)}_{\mu\nu} + E^{(D)}_{\mu\nu} + \mathcal{E}^{(D-1)}_{\mu\nu} + \kappa^4_D \pi_{\mu\nu} + \kappa^2_{D-1}\tau_{\mu\nu} + \Lambda_{D-1} g_{\mu\nu}, \tag{3.21}$$

where

$$\begin{aligned}\pi_{\mu\nu} &\equiv \frac{1}{4}\left\{\tau_{\mu\lambda}\tau^\lambda_\nu - \frac{1}{D-2}\tau\tau_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\left(\tau^{\alpha\beta}\tau_{\alpha\beta} - \frac{1}{D-2}\tau^2\right)\right\}, \\ \mathcal{E}^{(D-1)}_{\mu\nu} &\equiv \frac{\kappa^4_D (D-3)}{4(D-2)}\tau_p \times \left[\tau_{\mu\nu} + \left(g_k + \frac{1}{2}\tau_p\right) g_{\mu\nu}\right], \end{aligned} \tag{3.22}$$

and

$$\begin{aligned}\kappa^2_{D-1} &= \frac{D-3}{4(D-2)} g_k \kappa^4_D, \\ \Lambda_{D-1} &= \frac{D-3}{8(D-2)} g^2_k \kappa^4_D. \end{aligned} \tag{3.23}$$

For a perfect fluid,

$$\tau_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p g_{\mu\nu}, \tag{3.24}$$

where u_μ is the four-velocity of the fluid, we find that

$$\pi_{\mu\nu} = \frac{D-3}{4(D-2)}\rho \times \left[(\rho + p)\, u_\mu u_\nu - \left(p + \frac{1}{2}\rho\right) g_{\mu\nu}\right]. \tag{3.25}$$

Note that in writing eqs. (3.21)–(3.25), without causing any confusion, we had dropped the super indices (I).

JHEP10(2009)095

3.2.2 Matter field equations on the two branes

On the other hand, the I-th brane, localized on the surface $\Phi_I(x) = 0$, divides the spacetime into two regions, one with $\Phi_I(x) > 0$ and the other with $\Phi_I(x) < 0$. Since the field equations are the second-order differential equations, the matter fields have to be at least continuous across this surface, although in general their first-order directives are not. Introducing the Heaviside function, defined as

$$H(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0, \end{cases} \tag{3.26}$$

in the neighborhood of $\Phi_I(x) = 0$ we can write the matter fields in the form,

$$F(x) = F^+(x) H(\Phi_I) + F^-(x) [1 - H(\Phi_I)], \tag{3.27}$$

where $F \equiv \{\phi,\ \psi_\pm,\ \xi_\pm,\ B\}$, and F^+ (F^-) is defined in the region $\Phi_I > 0$ ($\Phi_I < 0$). Then, we find that

$$\begin{aligned} F_{,a}(x) &= F^+_{,a}(x) H(\Phi_I) + F^-_{,a}(x) [1 - H(\Phi_I)], \\ F_{,ab}(x) &= F^+_{,ab}(x) H(\Phi_I) + F^-_{,ab}(x) [1 - H(\Phi_I)] + [F_{,a}]^- \frac{\partial \Phi_I(x)}{\partial x^b} \delta(\Phi_I), \end{aligned} \tag{3.28}$$

where $[F_{,a}]^-$ is defined as that in eq. (3.18). Projecting $F_{,a}$ onto n^a and $e^a_{(\mu)}$ directions, we find

$$F_{,a} = F_{,\mu} e^{(\mu)}_a - F_{,n} n_a, \tag{3.29}$$

where

$$F_{,n} \equiv n^a F_{,a}, \quad F_{,\mu} \equiv e^a_{(\mu)} F_{,a}. \tag{3.30}$$

Then, we have

$$\begin{aligned} [F_{,a}]^- n^a &= [F_{,n}]^-, \\ [F_{,a}]^- e^a_{(\mu)} &= 0. \end{aligned} \tag{3.31}$$

Inserting eqs. (3.29)–(3.31) into eq. (3.28), we find

$$F_{,ab}(x) = F^+_{,ab}(x) H(\Phi_I) + F^-_{,ab}(x) [1 - H(\Phi_I)] - [F_{,n}]^- n_a n_b N_I\, \delta(\Phi_I), \tag{3.32}$$

where $N_I \equiv \sqrt{\left|\Phi_{I,c} \Phi_I^{;c}\right|}$, and

$$n_a = \frac{1}{N_I} \frac{\partial \Phi_I(x)}{\partial x^a}. \tag{3.33}$$

Substituting eq. (3.32) into eqs. (3.5)–(3.8), we find that the matter field equations on the branes read,

$$\left[\phi^{(I)}_{,n}\right]^- = -\Psi^{(I)} \left(2\kappa_D^2 \epsilon_I \frac{\partial V^{(I)}_{D-1}}{\partial \phi} + \sigma^{(I)}_\phi\right), \tag{3.34}$$

$$\left[\psi^{(I)}_{\pm,n}\right]^- = -\Psi^{(I)} \left(2\kappa_D^2 \epsilon_I \frac{\partial V^{(I)}_{D-1}}{\partial \psi_\pm} + \sigma^{(I)}_{\psi_\pm}\right), \tag{3.35}$$

JHEP10(2009)095

$$\left[\xi^{(I)}_{\pm,n}\right]^{-} = -\frac{\Psi^{(I)}}{2\alpha_{\pm}} e^{\sqrt{\frac{8}{d_{\pm}}}\psi_{\pm}-2\xi_{\pm}} \times \sum_{I=1}^{2}\left(2\kappa_D^2\epsilon_I\frac{\partial V^{(I)}_{D-1}}{\partial\xi_{\pm}} + \sigma^{(I)}_{\xi_{\pm}}\right), \tag{3.36}$$

$$\left[H^{(I)}_{nab}\right]^{-} = -\Psi^{(I)}\,\sigma^{(I)}_{ab}, \tag{3.37}$$

where

$$H_{nab} \equiv H_{cab}n^c, \quad \Psi^{(I)} \equiv \frac{1}{N_I}\sqrt{\left|\frac{g^{(I)}_{D-1}}{g_D}\right|}. \tag{3.38}$$

This completes our general description for $(D + d_+ + d_-)$-dimensional spacetimes of string theory with two orbifold branes.

4 10-dimensional spacetimes and brane cosmology

In this section, we restrict ourselves to the 10-dimensional spacetimes of string theory with $D = 5$ and $d_+ + d_- = 5$. It can be shown that the general metric for the five-dimensional spacetime with a 3-dimensional spatial space that is homogeneous, isotropic, and independent of time must take the form [5],

$$ds_5^2 = g_{ab}dx^a dx^b = g_{MN}dx^M dx^N - e^{2\omega(x^M)}d\Sigma_k^2, \tag{4.1}$$

where $M,\ N = 0, 1$. Choosing the conformal gauge,

$$g_{00} = g_{11}, \quad g_{01} = 0, \tag{4.2}$$

we find that the five-dimensional metric finally takes the form,

$$ds_5^2 = e^{2\sigma(t,y)}\left(dt^2 - dy^2\right) - e^{2\omega(t,y)}d\Sigma_k^2. \tag{4.3}$$

It should be noted that metric (4.3) is still subjected to the gauge freedom,

$$t = f(t' + y') + g(t' - y'), \quad y = f(t' + y') - g(t' - y'), \tag{4.4}$$

where $f(t' + y')$ and $g(t' - y')$ are arbitrary functions of their indicated arguments.

It should be noted that in [48] comoving branes were considered, and it was claimed that the gauge freedom of eq. (4.4) can always bring the two branes at rest (comoving). However, this excludes colliding branes [5, 24, 33]. In this paper, we shall leave this possibility open.

4.1 Field equations outside the two branes

To have the problem tractable, in the rest of this paper, we shall turn off the flux, i.e.,

$$\hat{B}_{CD} = 0, \tag{4.5}$$

so that

$$\xi_{\pm} = 0, \quad \alpha_{\pm} = 0, \quad \gamma_{\pm} = 0. \tag{4.6}$$

JHEP10(2009)095

Then, it can be shown that outside the two branes the field equations (3.1) have four independent components, which can be cast in the form,

$$\begin{aligned}
\omega_{,tt} + \omega_{,t}\left(\omega_{,t} - 2\sigma_{,t}\right) + & \\
+\omega_{,yy} + \omega_{,y}\left(\omega_{,y} - 2\sigma_{,y}\right) &= -\frac{1}{6}\left(\phi_{,t}^2 + \phi_{,y}^2 + \psi_{+,t}^2 + \psi_{+,y}^2 + \psi_{-,t}^2 + \psi_{-,y}^2\right), \quad (4.7)\\
2\sigma_{,tt} + \omega_{,tt} - 3\omega_{,t}{}^2 + & \\
-\left(2\sigma_{,yy} + \omega_{,yy} - 3\omega_{,y}{}^2\right) - 4ke^{2(\sigma-\omega)} &= -\frac{1}{2}\left(\phi_{,t}^2 - \phi_{,y}^2 + \psi_{+,t}^2 - \psi_{+,y}^2 + \psi_{-,t}^2 - \psi_{-,y}^2\right), \quad (4.8)\\
\omega_{,ty} + \omega_{,t}\omega_{,y} - \left(\sigma_{,t}\omega_{,y} + \sigma_{,y}\omega_{,t}\right) &= -\frac{1}{6}\left(\phi_{,t}\phi_{,y} + \psi_{+,t}\psi_{+,y} + \psi_{-,t}\psi_{-,y}\right), \quad (4.9)\\
\omega_{,tt} + 3\omega_{,t}{}^2 + & \\
-\left(\omega_{,yy} + 3\omega_{,y}{}^2\right) + 2ke^{2(\sigma-\omega)} &= \frac{1}{3}e^{2\sigma}V_5. \quad (4.10)
\end{aligned}$$

On the other hand, the Klein-Gordon equations (3.10) and (3.11) take the form,

$$\phi_{,tt} + 3\phi_{,t}\omega_{,t} - \left(\phi_{,yy} + 3\phi_{,y}\omega_{,y}\right) = -\sqrt{\frac{2}{3}}e^{2\sigma}V_5, \quad (4.11)$$

$$\psi_{+,tt} + 3\psi_{+,t}\omega_{,t} - \left(\psi_{+,yy} + 3\psi_{,y}\omega_{,y}\right) = e^{2\sigma}V_5, \quad (4.12)$$

$$\psi_{-,tt} + 3\psi_{-,t}\omega_{,t} - \left(\psi_{-,yy} + 3\psi_{,y}\omega_{,y}\right) = \sqrt{\frac{2}{3}}e^{2\sigma}V_5, \quad (4.13)$$

with

$$V_5 = e^{\sqrt{\frac{2}{3}}\phi}\left(\beta_+ e^{-\psi_+} + \beta_- e^{-\sqrt{\frac{2}{3}}\psi_-}\right). \quad (4.14)$$

4.2 Field equations on the two branes

Eqs. (4.7) - (4.12) are the field equations that are valid in between the two orbifold branes, $y_2(t_2) < y < y_1(t_1)$, where $y = y_I(t_I)$ denote the locations of the two branes. The proper distance between the two branes is given by

$$\mathcal{D}(t) = \int_{y_2}^{y_1} e^{\sigma(t,y)}dy. \quad (4.15)$$

On each of the two branes, the metric reduces to

$$ds_5^2\big|_{M_4^{(I)}} = g_{\mu\nu}^{(I)}d\xi_{(I)}^{\mu}d\xi_{(I)}^{\nu} = d\tau_I^2 - a^2\left(\tau_I\right)d\Sigma_k^2, \quad (4.16)$$

where $\xi_{(I)}^{\mu} \equiv \{\tau_I, r, \theta, \varphi\}$, and τ_I denotes the proper time of the I-th brane, defined by

$$\begin{aligned}
d\tau_I &= e^{\sigma[t_I(\tau_I), y_I(\tau_I)]}\sqrt{1 - \left(\frac{\dot{y}_I}{\dot{t}_I}\right)^2}\, dt_I,\\
a\left(\tau_I\right) &\equiv e^{\omega[t_I(\tau_I), y_I(\tau_I)]}, \quad (4.17)
\end{aligned}$$

with $\dot{y}_I \equiv dy_I/d\tau_I$, etc. For the sake of simplicity and without of causing any confusion, from now on we shall drop all the indices "I", unless some specific attention is needed.

Then, the normal vector n_a and the tangential vectors $e^a_{(\mu)}$ are given, respectively, by

$$\begin{aligned}
n_a &= e^{2\sigma}\left(-\dot{y}\delta^t_a + \dot{t}\delta^y_a\right),\\
n^a &= -\left(\dot{y}\delta^a_t + \dot{t}\delta^a_y\right),\\
e^a_{(\tau)} &= \dot{t}\delta^a_t + \dot{y}\delta^a_y, \quad e^a_{(r)} = \delta^a_r,\\
e^a_{(\theta)} &= \delta^a_\theta, \quad e^a_{(\varphi)} = \delta^a_\varphi.
\end{aligned} \tag{4.18}$$

Then, it can be shown that

$$\begin{aligned}
\mathcal{G}^{(5)}_{\mu\nu} &= \mathcal{G}^{(5)}_\tau \delta^\tau_\mu \delta^\tau_\nu - \mathcal{G}^{(5)}_\theta \delta^m_\mu \delta^n_\nu g_{mn},\\
E^{(5)}_{\mu\nu} &= E^{(5)}\left(3\delta^\tau_\mu\delta^\tau_\nu - \delta^m_\mu\delta^n_\nu g_{mn}\right),
\end{aligned} \tag{4.19}$$

where

$$\begin{aligned}
\mathcal{G}^{(5)}_\tau &\equiv \frac{1}{3}e^{-2\sigma}\left(\phi^2_{,t} - \phi^2_{,y} + \psi^2_{+,t} - \psi^2_{+,y} + \psi^2_{-,t} - \psi^2_{-,y}\right)\\
&\quad - \frac{5}{24}\left[(\nabla\phi)^2 + (\nabla\psi_+)^2 + (\nabla\psi_-)^2\right] + \frac{1}{4}V_5,\\
\mathcal{G}^{(5)}_\theta &\equiv \frac{1}{3}\left[\phi^2_{,n} + \psi^2_{+,n} + \psi^2_{-,n}\right] + \frac{5}{24}\left[(\nabla\phi)^2 + (\nabla\psi_+)^2 + (\nabla\psi_-)^2\right] - \frac{1}{4}V_5,\\
E^{(5)} &\equiv \frac{1}{6}e^{-2\sigma}\Big[(\sigma_{,tt} - \omega_{,tt}) - (\sigma_{,yy} - \omega_{,yy}) + ke^{2(\sigma-\omega)}\Big],
\end{aligned} \tag{4.20}$$

with $\phi_{,n} \equiv n^a\nabla_a\phi$. Then, it can be shown that the four-dimensional field equations on each of the two branes take the form,

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho + \tau_p) + \frac{1}{3}\Lambda + \frac{1}{3}\mathcal{G}^{(5)}_\tau + E^{(5)} + \frac{2\pi G}{3\rho_\Lambda}(\rho + \tau_p)^2, \tag{4.21}$$

$$\begin{aligned}
\frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p - 2\tau_p) + \frac{1}{3}\Lambda - E^{(5)} - \frac{1}{6}\left(\mathcal{G}^{(5)}_\tau + 3\mathcal{G}^{(5)}_\theta\right)\\
&\quad - \frac{2\pi G}{3\rho_\Lambda}\left[\rho(2\rho + 3p) + (\rho + 3p - \tau_p)\tau_p\right],
\end{aligned} \tag{4.22}$$

where $H \equiv \dot{a}/a,$, $\Lambda \equiv \Lambda_4$ and $G \equiv G_4$.

On the other hand, from eqs. (3.34) and (3.35), we find that

$$\left[\phi^{(I)}_{,n}\right]^- = -\left(2\kappa^2_5\epsilon_I\frac{\partial V^{(I)}_4}{\partial\phi} + \sigma^{(I)}_\phi\right)\Psi, \tag{4.23}$$

$$\left[\psi^{(I)}_{\pm,n}\right]^- = -\left(2\kappa^2_5\epsilon_I\frac{\partial V^{(I)}_4}{\partial\psi_\pm} + \sigma^{(I)}_{\psi_\pm}\right)\Psi. \tag{4.24}$$

5 Radion stability and radion mass

In the studies of branes, an important issue is the radion stability. In this section, we shall address this problem. For such a purpose, let us consider the 5-dimensional static metric with a 4-dimensional Poincaré symmetry, which is given by eq. (4.3) with $k = 0$ and $\sigma(y) = \omega(y)$, that is,

$$ds^2_5 = e^{2\sigma(y)}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dy^2\right). \tag{5.1}$$

JHEP10(2009)095

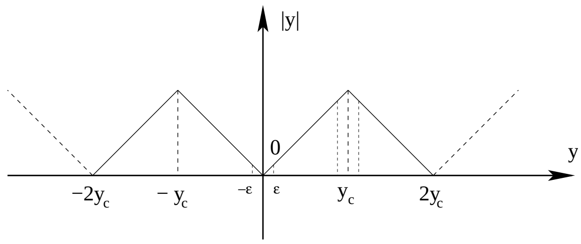

Figure 1. The function $|y|$ appearing in the metric eq. (5.2).

Then, we find that the corresponding solutions are given by,

$$\begin{aligned}
\sigma(y) &= \frac{1}{3} ln \left(\frac{|y| + y_0}{L} \right), \\
\phi(y) &= c_1 ln \left(\frac{|y| + y_0}{L} \right) + \phi_0, \\
\psi_+(y) &= c_2 ln \left(\frac{|y| + y_0}{L} \right) + \psi_+^0, \\
\psi_-(y) &= \sqrt{\frac{3}{2}} c_2 \ln \left(\frac{|y| + y_0}{L} \right) + \psi_-^0,
\end{aligned} \tag{5.2}$$

where $c_1, y_0,\ L,\ \ \sigma_0,\ \phi_0$, and ψ_+^0 are all arbitrary constants, and

$$\begin{aligned}
c_2 &= \pm \sqrt{\frac{2\left(8 - 3c_1^2\right)}{15}}, \\
\psi_-^0 &= \sqrt{\frac{3}{2}} \left(\psi_0 - \ln \left(\frac{-\beta_+}{\beta_-} \right) \right).
\end{aligned} \tag{5.3}$$

The function $|y|$ is defined as in figure 1.

Then, it can be shown that the above solution satisfies the gravitational and matter field equations outside the branes, eqs. (4.7)–(4.12). On the other hand, to show that it also satisfies the field equations on the branes, given by eqs. (4.21)–(4.22) and eqs. (4.23)–(4.24), we first note that the normal vector $n^a_{(I)}$ to the I-th brane is given by

$$n^a_{(I)} = -\epsilon_y^{(I)} e^{-\sigma(y_I)} \delta^a_y, \tag{5.4}$$

and that

$$\begin{aligned}
\dot{t} &= e^{-\sigma(y_I)}, \ \ \dot{y} = 0, \\
\mathcal{G}^{(5)}_\tau &= -\mathcal{G}^{(5)}_\theta = -\frac{2}{9L^2} \left(\frac{L}{y_I + y_0} \right)^{\frac{8}{3}},
\end{aligned} \tag{5.5}$$

where $y_1 = y_c > 0$ and $y_2 = 0$. Inserting the above into eqs. (4.21) and (4.22), and considering the fact that $H = 0$ we find that these two equations are satisfied for $\tau^{(I)}_{\mu\nu} = 0$, provided that the tension $\tau_p^{(I)}$ defined by eq. (3.4) satisfies the relation,

$$\left(\tau^{(I)}_{(\phi,\psi_\pm)} + 2 \rho_\Lambda{}^{(I)} \right)^2 = \frac{\rho_\Lambda^{(I)}}{9\pi G_4 L^2} \left(\frac{L}{y_I + y_0} \right)^{8/3}, \tag{5.6}$$

where $\rho_\Lambda^{(I)}$ denotes the corresponding energy density of the effective cosmological constant on the I-th brane, defined as $\rho_\Lambda^{(I)} = \Lambda^{(I)}/(8\pi G)$. On the other hand, from eqs. (4.23) and (4.24) we find that

$$\frac{\partial V_4^{(I)}}{\partial \phi} = \frac{c_1 \epsilon_I}{\kappa_5^2 (y_I + y_0)}, \tag{5.7}$$

$$\frac{\partial V_4^{(I)}}{\partial \psi_+} = \frac{c_2 \epsilon_I}{\kappa_5^2 (y_I + y_0)}, \tag{5.8}$$

$$\frac{\partial V_4^{(I)}}{\partial \psi_-} = -\frac{\sqrt{3} c_2 \epsilon_I}{\sqrt{2} \kappa_5^2 (y_I + y_0)}, \tag{5.9}$$

To study the radion stability, it is found convenient to introduce the proper distance Y, defined by

$$Y = \frac{3L}{4} \left(\left(\frac{y_I + y_0}{L} \right)^{4/3} - \left(\frac{y_0}{L} \right)^{4/3} \right). \tag{5.10}$$

Then, in terms of Y, the static solution (5.1) can be written as

$$ds_5^2 = e^{-2A(Y)} \eta_{\mu\nu} dx^\mu dx^\nu - dY^2, \tag{5.11}$$

with

$$\begin{aligned} A(Y) &= -\frac{1}{4} \ln \left(\frac{4 (|Y| + Y_0)}{3L} \right), \\ \phi(Y) &= \frac{3}{4} c_1 \ln \left(\frac{4 (|Y| + Y_0)}{3L} \right) + \phi_0, \\ \psi_+(Y) &= \frac{3}{4} c_2 \ln \left(\frac{4 (|Y| + Y_0)}{3L} \right) + \psi_+^0, \\ \psi_-(Y) &= \sqrt{\frac{27}{32}}\, c_2 \ln \left(\frac{4 (|Y| + Y_0)}{3L} \right) + \psi_-^0, \end{aligned} \tag{5.12}$$

where

$$Y_0 = \frac{3L}{4} \left(\frac{y_0}{L} \right)^{4/3}. \tag{5.13}$$

Following [49], let us consider a massive scalar field Φ with the actions,

$$\begin{aligned} S_b &= \int d^4x \int_0^{Y_c} dY \sqrt{-g_5} \left((\nabla \Phi)^2 - M^2 \Phi^2 \right), \\ S_I &= -\alpha_I \int_{M_4^{(I)}} d^4x \sqrt{-g_4^{(I)}} \left(\Phi^2 - v_I^2 \right)^2, \end{aligned} \tag{5.14}$$

where α_I and v_I are real constants. In this paper we introduce this field as a purely phenomenological field. However, the introduction of such an action can be given string theoretic justification as coming from the massive breathing modes of sphere reduction of string theory effective actions [41]. Since the geometry of our internal spaces is left arbitrary, it is presumably possible to obtain the Goldberger-Wise field from our compactification. This is a question that we plan to return in the future.

Then, it can be shown that, in the background of eq. (5.11), the massive scalar field Φ satisfies the following Klein-Gordon equation

$$\Phi'' - 4A'\Phi' - M^2\Phi = \sum_{I=1}^{2} 2\alpha_I \Phi\left(\Phi^2 - v_I^2\right)\delta(Y - Y_I). \tag{5.15}$$

Integrating the above equation in the neighborhood of the I-th brane, we find that

$$\left.\frac{d\Phi(Y)}{dY}\right|_{Y_I-\epsilon}^{Y_I+\epsilon} = 2\alpha_I \Phi_I\left(\Phi_I^2 - v_I^2\right), \tag{5.16}$$

where $\Phi_I \equiv \Phi(Y_I)$. Since

$$\begin{aligned}
\lim_{Y\to Y_c^+}\frac{d\Phi(Y)}{dY} &= -\lim_{Y\to Y_c^-}\frac{d\Phi(Y)}{dY} \equiv -\Phi'(Y_c),\\
\lim_{Y\to 0^-}\frac{d\Phi(Y)}{dY} &= -\lim_{Y\to 0^+}\frac{d\Phi(Y)}{dY} \equiv -\Phi'(0),
\end{aligned} \tag{5.17}$$

we find that the conditions (5.16) can be written in the forms,

$$\Phi'(Y_c) = -\alpha_1\Phi_1\left(\Phi_1^2 - v_1^2\right), \tag{5.18}$$

$$\Phi'(0) = \alpha_2\Phi_2\left(\Phi_2^2 - v_2^2\right). \tag{5.19}$$

Inserting the above solution back to the actions (5.14), and then integrating them with respect to Y, we obtain the effective potential for the radion Y_c,

$$\begin{aligned}
V_\Phi(Y_c) &\equiv -\int_{0+\epsilon}^{Y_c-\epsilon} dY\sqrt{-g_5}\left((\nabla\Phi)^2 - M^2\Phi^2\right)\\
&\quad+\sum_{I=1}^{2}\alpha_I\int_{Y_I-\epsilon}^{Y_I+\epsilon} dY\sqrt{-g_4^{(I)}}\left(\Phi^2 - v_I^2\right)^2\times\delta(Y - Y_I)\\
&= e^{-4A(Y)}\Phi(Y)\Phi'(Y)\Big|_0^{Y_c} + \sum_{I=1}^{2}\alpha_I\left(\Phi_I^2 - v_I^2\right)^2 e^{-4A(Y_I)}.
\end{aligned} \tag{5.20}$$

For the background solution given by eq. (5.12), one find that in the region $0 < Y < Y_c$, eq. (5.15) reads,

$$\frac{d^2\Phi}{dz^2} + \frac{1}{z}\frac{d\Phi}{dz} - \Phi = 0, \tag{5.21}$$

where $z \equiv M(Y + Y_0)$. Eq. (5.21) has the general solution,

$$\Phi = aI_0(z) + bK_0(z), \tag{5.22}$$

where $I_0(z)$ and $K_0(z)$ denote the modified Bessel function of the first and second kind, respectively [50]. In the limit that α_I's are very large [49], eqs. (5.18) and (5.19) show that there are solutions only when $\Phi(0) \simeq v_2$ and $\Phi(Y_c) \simeq v_1$, that is,

$$v_1 \simeq aI_0^c + bK_0^c, \tag{5.23}$$

$$v_2 \simeq aI_0^0 + bK_0^0, \tag{5.24}$$

JHEP10(2009)095

where $z_c = M(Y_c + Y_0)$, $z_0 = MY_0$, $I_0^i \equiv I_0(z_i)$ and $K_0^i \equiv K_0(z_i)$. eqs. (5.23) and (5.24) have the solution,

$$
\begin{aligned}
a &\simeq \frac{1}{\Delta}\left(v_1 K_0^0 - v_2 K_0^c\right), \\
b &\simeq \frac{1}{\Delta}\left(v_2 I_0^c - v_1 I_0^0\right),
\end{aligned} \tag{5.25}
$$

where

$$
\Delta \equiv I_0^c K_0^0 - I_0^0 K_0^c. \tag{5.26}
$$

Inserting eqs. (5.22) and (5.25) into eq. (5.20), we find that

$$
\begin{aligned}
V_\Phi(Y_c) \simeq \frac{4}{3L\Delta}\Big\{ & v_1 z_c \left[v_1\left(I_0^0 K_1^c + I_1^c K_0^0\right) - v_2\left(I_0^c K_1^c + I_1^c K_0^c\right)\right] \\
& + v_2 z_0 \left[v_2\left(I_0^c K_1^0 + I_1^0 K_0^c\right) - v_1\left(I_0^0 K_1^0 + I_1^0 K_0^0\right)\right]\Big\}.
\end{aligned} \tag{5.27}
$$

To further study the potential, let us consider two different limits, $z_0 \gg 1$ and $z_0 \ll 1$. With all these free parameters at hand, it is not difficult to see that the mass of the radion should be also in the order of TeV, as we obtained previously in both string [28] and M theory [5].

5.1 $z_0 \gg 1$

When $z_0 \gg 1$, we have $z_c = z_0 + MY_c \gg 1$. Then, we find

$$
\begin{aligned}
I_0(z) &\simeq I_1(z) \simeq \sqrt{\frac{1}{2\pi z}} e^{z}, \\
K_0(z) &\simeq K_1(z) \simeq \sqrt{\frac{\pi}{2z}} e^{-z}.
\end{aligned} \tag{5.28}
$$

Inserting the above expressions into eq. (5.27), we obtain

$$
V_\Phi(Y_c) \simeq \frac{4z_0}{3L\sinh(MY_c)}\left\{\left({v_1}^2 + {v_2}^2\right)\cosh(MY_c) - 2v_1 v_2\right\}, \tag{5.29}
$$

which has a minimum at

$$
Y_c^{min.} = \frac{1}{M}\cosh^{-1}\left(\frac{{v_1}^2 + {v_2}^2}{2v_1 v_2}\right), \tag{5.30}
$$

where

$$
\begin{aligned}
\left.\frac{\partial^2 V_\Phi(Y_c)}{\partial {Y_c}^2}\right|_{Y_c = Y_c^{min.}} &\simeq \left(\frac{16 z_0 M^2}{3L}\right)\frac{(v_1 v_2)^2}{\left|v_1^2 - v_2^2\right|} > 0, \\
V_\Phi(Y_c) &\simeq \begin{cases} \infty, & Y_c = 0, \\ \infty, & Y_c = \infty. \end{cases}
\end{aligned} \tag{5.31}
$$

Figure 2 shows the potential schematically, from which we can see that it always has a minimum at a finite and non-zero value of Y_c. Therefore, in the present setup, the radion is stable in the limit $M \gg 1/Y_0$.

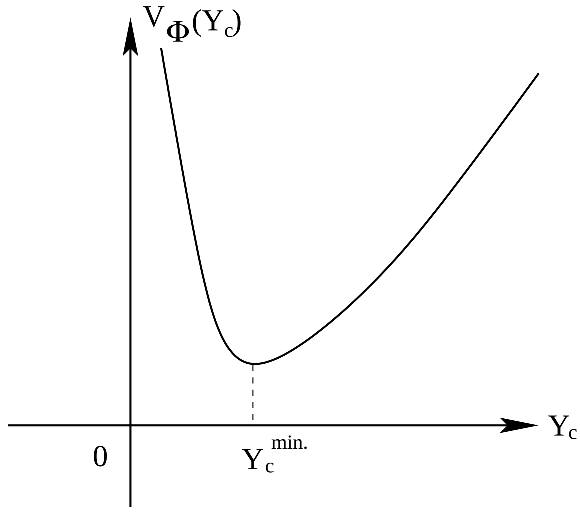

Figure 2. The potential $V_\Phi\left(Y_c\right)$ given by eq. (5.29) for $M \gg 1/Y_0$.

To calculate the corresponding radion mass, we need to know the precise relation between Y_c and the radion scalar φ. Following [5, 49], we find that

$$\varphi=\left(\frac{12}{\kappa_5^2}\int_0^{Y_c}e^{-2A}dY\right)^{1/2}=\sqrt{6LM_5^3}\times\left\{\left(\frac{4\left(Y_c+Y_0\right)}{3L}\right)^{3/2}-\left(\frac{4Y_0}{3L}\right)^{3/2}\right\}^{1/2}. \quad (5.32)$$

Then, we obtain that

$$m_\varphi^2\equiv\left.\frac{\partial^2V_\Phi\left(Y_c\right)}{2\partial\varphi^2}\right|_{Y_c=Y_c^{min.}}=\frac{M^2}{M_5^3}\left(\frac{16Y_0}{27L}\right)^{1/2}\times\frac{\left(v_1v_2\right)^2}{\left|v_1^2-v_2^2\right|}\cosh^{-1}\left(\frac{{v_1}^2+{v_2}^2}{2v_1v_2}\right), \quad (5.33)$$

where $M_5^3=M_{10}^8V_{d_+}V_{d_-}$, as can be seen from eqs. (2.4) and (2.17).

5.2 $z_0 \ll 1$

When $z \ll 1$, we find

$$\begin{aligned} I_0(z)&\simeq 1, & I_1(z)&\simeq\frac{z}{4},\\ K_0(z)&\simeq-\ln(z), & K_1(z)&\simeq\frac{1}{z}. \end{aligned} \quad (5.34)$$

Then, eq. (5.27) reduces to

$$V_\Phi\left(Y_c\right)\simeq\frac{v_1-v_2}{3LY_c}\left\{\left(v_1-v_2\right)\left(4-z_0^2\ln\left(z_0\right)\right)Y_0+z_0^2\left(v_2-2v_1\ln\left(z_0\right)\right)Y_c\right\}, \quad (5.35)$$

for $Y_c \ll Y_0$. Figure 3 shows the potential schematically, from which we can see that it has non-minimum. That is, the radion is not stable for $M \ll 1/Y_0$. Combining it with last case, we find that there must exist a critical M_c, for which the radion is stable when $M > M_c > 0$, and not stable when $M < M_c$.

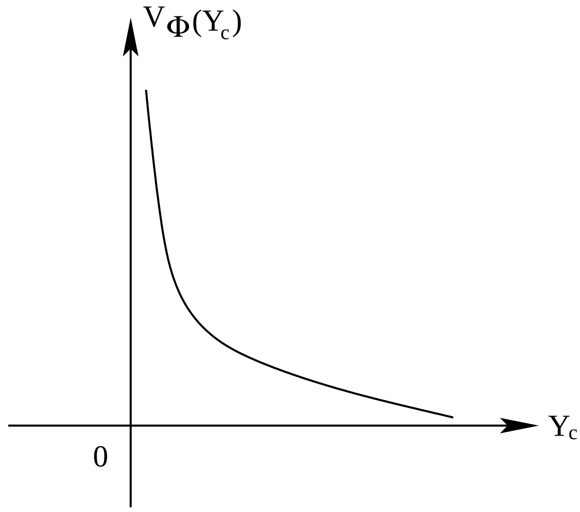

Figure 3. The potential $V_{\Phi}\left(Y_c\right)$ given by eq. (5.35) for $M \ll 1/Y_0$.

6 Localization of gravity and 4D effective Newtonian potential

To study the localization of gravity and the four-dimensional effective gravitational potential, in this section let us consider small fluctuations h_{ab} of the 5-dimensional static metric with a 4-dimensional Poincaré symmetry, given by eqs. (5.1) in its conformally flat form.

6.1 Tensor perturbations and the KK towers

Since such tensor perturbations are not coupled with scalar ones [51], without loss of generality, we can set the perturbations of the scalar fields to zero, i.e., $\delta\phi_n = 0$. We shall choose the gauge [7, 17]

$$h_{ay} = 0, \quad h^{\lambda}_{\lambda} = 0 = \partial^{\lambda} h_{\mu\lambda}. \tag{6.1}$$

Then, it can be shown that [52]

$$\begin{aligned}
\delta G^{(5)}_{ab} &= -\frac{1}{2}\Box_5 h_{ab} - \frac{3}{2}\left\{\left(\partial_c\sigma\right)\left(\partial^c h_{ab}\right) - 2\left[\Box_5\sigma + \left(\partial_c\sigma\right)\left(\partial^c\sigma\right)\right] h_{ab}\right\},\\
\kappa_5^2 \delta T^{(5)}_{ab} &= -\frac{1}{4} h_{ab}\left(\sum_{n=1}\left(\nabla\phi_n\right)^2 - 2V_5\right),\\
\delta T^{(4)}_{\mu\nu} &= \left(\tau_p + \lambda\right) h_{\mu\nu},
\end{aligned} \tag{6.2}$$

where $\Box_5 \equiv \eta^{ab}\partial_a\partial_b$ and $\left(\partial_c\sigma\right)\left(\partial^c h_{ab}\right) \equiv \eta^{cd}\left(\partial_c\sigma\right)\left(\partial_d h_{ab}\right)$, with η^{ab} being the five-dimensional Minkowski metric. Substituting the above expressions into the gravitational field equations (3.1) with $D = 5$, we find that in the present case there is only one independent equation, given by

$$\Box_5 \tilde{h}_{\mu\nu} + \frac{3}{2}\left(\sigma'' + \frac{3}{2}\sigma'^2\right)\tilde{h}_{\mu\nu} = 0, \tag{6.3}$$

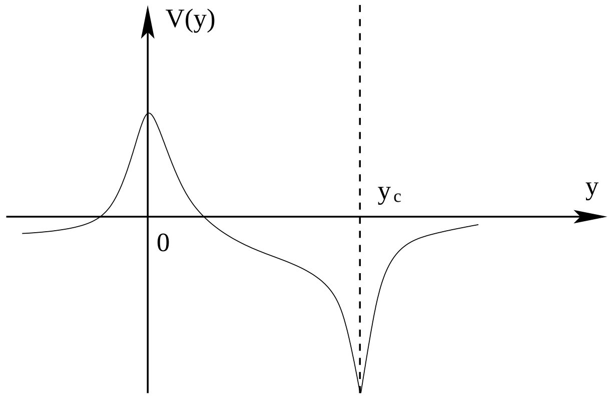

Figure 4. The potential defined by eq. (6.6).

where $h_{\mu\nu} \equiv e^{-3\sigma/2}\tilde{h}_{\mu\nu}$. Setting

$$\begin{aligned} \tilde{h}_{\mu\nu}(x,y) &= \hat{h}_{\mu\nu}(x)\psi(y), \\ \Box_5 &= \Box_4 - \nabla_y^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu - \partial_y^2, \\ \Box_4\hat{h}_{\mu\nu}(x) &= -m^2\hat{h}_{\mu\nu}(x), \end{aligned} \tag{6.4}$$

we find that eq. (6.3) takes the form of the schrödinger equation,

$$\left(-\nabla_y^2 + V\right)\psi = m^2\psi, \tag{6.5}$$

where

$$V \equiv \frac{3}{2}\left(\sigma'' + \frac{3}{2}\sigma'^2\right) = -\frac{1}{4\left(|y| + y_0\right)^2} + \frac{\delta(y)}{y_0} - \frac{\delta\left(y - y_c\right)}{y_c + y_0}. \tag{6.6}$$

From the above expression we can see clearly that the potential has a delta-function well at $y = y_c$, which is responsible for the localization of the graviton on this brane. In contrast, the potential has a delta-function barrier at $y = 0$, which makes the gravity delocalized on the $y = 0$ brane. Figure 4 shows the potential schematically.

Integration of eq. (6.5) in the neighbourhood of $y = 0$ and $y = y_c$ yields, respectively, the boundary conditions,

$$\lim_{y\to y_c^-} \psi'(y) = \frac{1}{2\left(y_c + y_0\right)} \lim_{y\to y_c^-} \psi(y), \tag{6.7}$$

$$\lim_{y\to 0^+} \psi'(y) = \frac{1}{2y_0} \lim_{y\to 0^+} \psi(y). \tag{6.8}$$

Note that in writing the above equations we had used the Z_2 symmetry of the wave function ψ.

Introducing the operators,

$$Q \equiv \nabla_y - \frac{3}{2}\sigma', \quad Q^\dagger \equiv -\nabla_y - \frac{3}{2}\sigma', \tag{6.9}$$

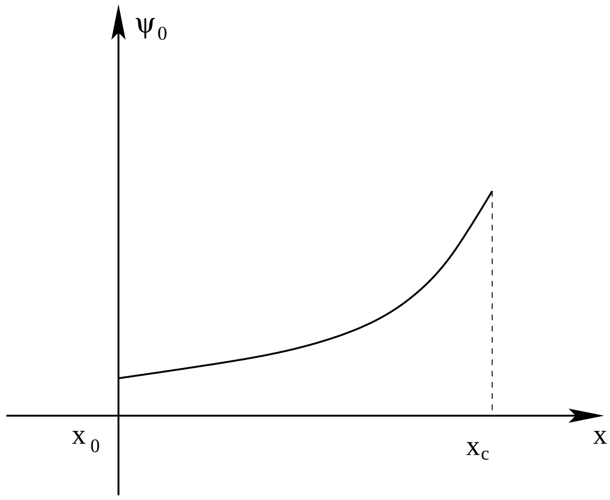

Figure 5. The zero-mode wavefunction given by eq. (6.11), from which we can see that the gravity is localized on the visible brane at $x = x_c$.

eq. (6.5) can be written in the form of a supersymmetric quantum mechanics problem,

$$Q^\dagger \cdot Q\psi = m^2\psi, \tag{6.10}$$

which, together with the boundary conditions (6.7) and (6.8), guarantees that the operator $Q^\dagger \cdot Q$ is Hermitian [5, 53]. Then, by the usual theorems from Quantum Mechanics [54], we can see that all eigenvalues m^2 are non-negative, and their corresponding wave functions $\psi_n(y)$ are orthogonal to each other and form a complete basis. Therefore, the background in the current setup is gravitationally stable.

6.1.1 Zero mode

The four-dimensional gravity is given by the existence of the normalizable zero mode, for which the corresponding wavefunction is given by

$$\psi_0(y) = N_0 \left(|y| + y_0\right)^{1/2}, \tag{6.11}$$

where N_0 is the normalization factor, defined as

$$N_0 = \sqrt{\frac{2}{y_c\,(y_c + 2y_0)}}. \tag{6.12}$$

Eq. (6.11) shows clearly that the wavefunction is increasing as y increases from 0 to y_c [cf. figure 5]. Therefore, the gravity is indeed localized near the $y = y_c$ brane.

6.1.2 Non-Zero modes

In order to have localized four-dimensional gravity, we require that the corrections to the Newtonian law from the non-zero modes, the KK modes, of eq. (6.5), be very small, so

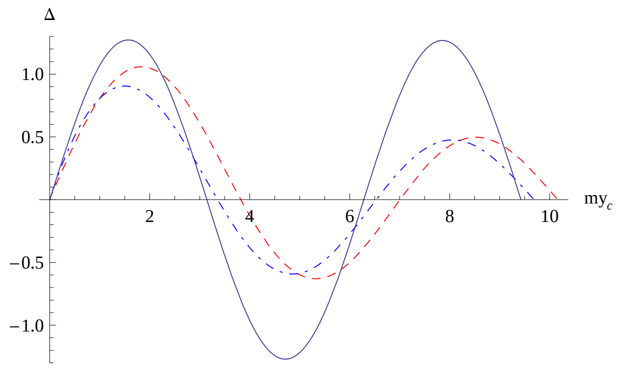

Figure 6. The re-scaled function of Δ defined by eq. (6.15), where the dashed, dot-dashed and solid lines are, respectively, for $\Delta\left(x_0 = 0.01\right)/35; \Delta\left(x_0 = 1.0\right)/0.5$ and $\Delta\left(x_0 = 1000\right)/0.005$.

x_0	$m_1 y_c$	$m_2 y_c$	$m_3 y_c$
0.01	3.82	7.01	10.16
1.0	3.36	6.53	9.69
1000	3.14	6.28	9.42

Table 1. The first three modes m_n $(n = 1, 2, 3)$ for $x_0 = 0.01,\ 1.0,\ 1000$, respectively.

that they will not lead to contradiction with observations. When $m \neq 0$, it can be shown that eq. (6.5) has the general solution,

$$\psi = x^{1/2}\left(cJ_0(x) + dY_0(x)\right), \tag{6.13}$$

where $x \equiv m\left(y + y_0\right)$, and $J_0(x)$ and $Y_0(x)$ are the Bessel functions of the first and second kind, respectively [50]. The integration constants c and d are determined from the boundary conditions, eqs. (6.7) and (6.8), which can now be cast in the form,

$$\begin{pmatrix} J_1\left(x_c\right) & Y_1\left(x_c\right) \\ J_1\left(x_0\right) & Y_1\left(x_0\right) \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = 0, \tag{6.14}$$

where $x_0 \equiv my_0$ and $x_c \equiv x_0 + my_c$. Clearly, it has no trivial solutions only when

$$\begin{aligned} \Delta\left(x_0, x_c\right) &\equiv J_1\left(x_c\right) Y_1\left(x_0\right) - J_1\left(x_0\right) Y_1\left(x_c\right) \\ &= 0. \end{aligned} \tag{6.15}$$

Figure 6 shows the function $\Delta\left(x_0, my_c\right)$ for $x_0 = 0.01,\ 1.0,\ 1000$, respectively. Note that in plotting these lines, properly rescaling toke place. From this figure, we find that the spectrum of the gravitational KK towers is discrete, and weakly depends on the specific values of x_0.

Table I shows the first three modes m_n $(n = 1, 2, 3)$ for $x_0 = 0.01,\ 1.0,\ 1000$, from which we can see that to find m_n it is sufficient to consider only the case where $x_0 \gg 1$.

When $x_0 \gg 1$ we find that $x_c = x_0 + my_c \gg 1$ and [50]

$$\begin{aligned} J_1(x) &\simeq \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{3}{4}\pi\right), \\ Y_1(x) &\simeq \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{3}{4}\pi\right). \end{aligned} \tag{6.16}$$

Inserting the above expressions into eq. (6.15), we obtain

$$\Delta = \frac{2}{\pi\sqrt{x_0 x_c}} \sin\left(m y_c\right), \tag{6.17}$$

whose roots are given by

$$m_n = \frac{n\pi}{y_c}, \quad (n = 1, 2, \ldots). \tag{6.18}$$

In particular, we have

$$m_1 \simeq 3.14 \times \left(\frac{10^{-19}\ \mathrm{m}}{y_c}\right)\ \mathrm{TeV} \simeq \begin{cases} 1\ \mathrm{TeV}, & y_c \simeq 10^{-19}\ \mathrm{m}, \\ 10^{-2}\ \mathrm{eV}, & y_c \simeq 10^{-5}\ \mathrm{m}, \\ 10^{-4}\ \mathrm{eV}, & y_c \simeq 10^{-3}\ \mathrm{m}. \end{cases} \tag{6.19}$$

It should be noted that the mass m_n calculated above is measured by the observer with the metric $\eta_{\mu\nu}$. However, since the warped factor $e^{\sigma(y)}$ is not one at $y = y_c$, the physical mass on the visible brane should be given by [7]

$$m_n^{\mathrm{obs}} = e^{-\sigma(y_c)} m_n = \left(\frac{y_c + y_0}{L}\right)^{1/3} m_n. \tag{6.20}$$

Without introducing any new hierarchy, we expect that $\left[(y_c + y_0)/L\right]^{1/3} \simeq \mathcal{O}(1)$. As a result, we have

$$m_n^{\mathrm{obs}} = \left(\frac{y_c + y_0}{L}\right)^{1/3} m_n \simeq m_n. \tag{6.21}$$

For each m_n that satisfies eq. (6.15), the wavefunction $\psi_n(y)$ is given by

$$\psi_n(y) = N_n x_n^{1/2} \left(\frac{J_0\left(x_n\right)}{J_1\left(x_{0,n}\right)} - \frac{Y_0\left(x_n\right)}{Y_1\left(x_{0,n}\right)}\right), \tag{6.22}$$

where

$$\begin{aligned} x_{0,n} &\equiv m_n y_0 \simeq n\pi\left(\frac{y_0}{y_c}\right), \\ x_n &\equiv m_n\left(y_0 + y\right) \simeq n\pi\left(\frac{y_0 + y}{y_c}\right). \end{aligned} \tag{6.23}$$

The normalization factor $N_n[\equiv N_n\left(m_n, y_c\right)]$ is determined by the condition,

$$\int_0^{y_c} \left|\psi_n(y)\right|^2 dy = 1. \tag{6.24}$$

Figures 7, 8 and 9 show $\psi_1\left(y\right),\ \psi_2\left(y\right)$ and $\psi_3\left(y\right)$ for $x_{0,1} = 100,\ 102,\ 104$, respectively.

JHEP10(2009)095

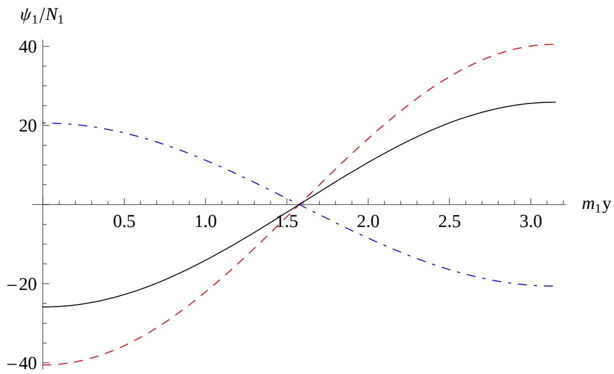

Figure 7. The wavefunction, $\psi_1\,(y)$, defined by eq. (6.22) vs $m_1 y$ where $y \in [0, y_c]$. The dashed, dot-dashed and solid lines are, respectively, for $x_{0,1} = 100,\ 102,\ 104$.

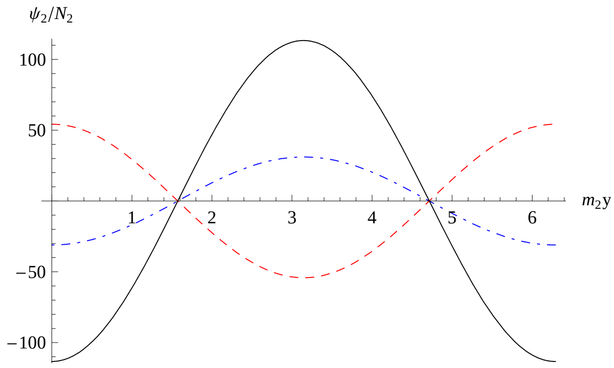

Figure 8. The wavefunction, $\psi_2\,(y)$, defined by eq. (6.22), vs $m_2 y$ where $y \in [0, y_c]$. The dashed, dot-dashed and solid lines are, respectively, for $x_{0,1} = 100,\ 102,\ 104$.

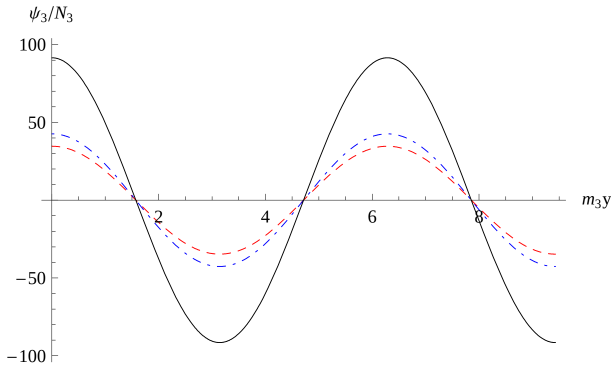

Figure 9. The wavefunction, $\psi_3\,(y)$, defined by eq. (6.22), vs $m_3 y$ where $y \in [0, y_c]$. The dashed, dot-dashed and solid lines are, respectively, for $x_{0,1} = 100,\ 102,\ 104$.

6.2 4D Newtonian potential and Yukawa corrections

To calculate the four-dimensional effective Newtonian potential and its corrections, let us consider two point-like sources of masses M_1 and M_2, located on the brane at $y = y_c$. Then, the discrete eigenfunction $\psi_n(z)$ of mass m_n has an Yukawa correction to the four-dimensional gravitational potential between the two particles [52, 55],

$$U(r) = G_4 \frac{M_1 M_2}{r} + \frac{M_1 M_2}{M_5^3 r} \sum_{n=1}^{\infty} e^{-m_n r} \left|\psi_n\left(y_c\right)\right|^2, \tag{6.25}$$

where $\psi_n(y_c)$ is given by eq. (6.22), with

$$x_{c,n} \equiv m_n\left(y_c + y_0\right) \simeq \frac{n\pi y_0}{y_c} + n\pi. \tag{6.26}$$

When $x_{0,1} = m_1 y_0 \gg 1$, we find that

$$N_n \simeq \frac{\cos\left(2m_n y_0\right)}{\sqrt{2n\pi y_0}},$$
$$\psi_n\left(y_c\right) \simeq (-1)^{n+1}\sqrt{\frac{2}{y_c}}. \tag{6.27}$$

Then, we obtain,

$$\left|\psi_n\left(y_c\right)\right|^2 \simeq 2M_{pl}\left(\frac{l_{pl}}{y_c}\right). \tag{6.28}$$

Clearly, by properly choosing y_c, the corrections of the 4-dimensional Newtonian potential due to the high order gravitational KK modes are negligible.

7 Conclusions

In this paper, we have systematically studied the possibility of implementing the RS1 scenario [7] into type II string theory on an S^1/Z_2 orbifold. In particular, in section II, starting with the Neveu-Schwarz/Neveu-Schwarz (NS/NS) sector, we have first compactified the $(D + d_+ + d_-)$-dimensional spacetime on two manifolds M_{d_+} and M_{d_-}, where the topologies of M_{d_+} and M_{d_-} are unspecified. As shown explicitly there, this particularly allows the dilaton and modulus fields to have non-zero potentials (masses), which is in contrast to the toroidal compactification considered previously [26–28, 31–33]. After reducing the action to an effective D-dimensional one, which is given by eq. (2.16) in the Einstein frame, we further compactify one of the $(D - 1)$ spatial dimensions on an S^1/Z_2 orbifold, by adding the brane actions (2.21). This completes the whole setup of the model to be studied in this paper. Lifting it to the original spacetime, the two orbifold branes become $(D + d_+ + d_- - 1)$-dimensional.

In section III, we have explicitly derived the corresponding gravitational and matter field equations both in the bulk and on the branes, by using the Gauss-Codacci and Lanczos equations. In section IV such developed formulas have been applied to cosmology by setting $D = 5 = d_+ + d_-$. In particular, the generalized Friedmann equations on the branes are given explicitly by eqs. (4.21) and (4.22).

In section V, in order to study the radion stability and radion mass, we have first derived the general static solutions with a 4-dimensional Poincaré symmetry. Then, using the Goldberger-Wise mechanism, we have studied the radion stability and shown explicitly that it is indeed stable in our current setup. The corresponding radion mass is given by

JHEP10(2009)095

eq. (5.33), from which we can see that the observational constraint $m_\varphi > 10^{-3}\ eV$ can be easily satisfied by properly choosing the free parameters presented in the model.

In section VI, we have studied the tensor perturbations, and shown explicitly that the background solution is gravitational stable, and the gravity is localized on the visible brane, as one can be seen clearly from figure 5. Due to the particular boundary conditions, the spectrum of the gravitational KK towers is discrete, and the corresponding masses can be well approximated by eq. (6.18), as one can see from figure 6 and table I. The mass gap $\Delta m \equiv m_1$ between the ground state and the first excited state can be in the order of TeV, while the high order Yukawa corrections to the 4-dimensional Newtonian potential, due to the high order KK modes, is exponentially suppressed, and can be negligible.

The above results strongly support our earlier conclusions obtained in the studies of orbifold branes in both the HW heterotic M theory [5, 25] and string theory [26–28]. In particular, in all these models the radion is stable, and the gravity is localized on the visible (TeV) branes, in contrast to the RS1 model [7], where the gravity is localized on the invisible brane. Our models are much more complicated than the RS1 model and involve several free parameters. By properly choosing them, the theory should be consistent with observational constraints, a subject that is under our current investigations. It would be also extremely interesting to find specific models in the current setup to explain the late cosmic acceleration of the universe [56].

Acknowledgments

One of the authors (AW) would like to thank K. Koyama, R. Maartens, A. Papazoglou, Y.-S. Song and D. Wands for valuable discussions. He also would also like to express his gratitude to the Institute of Cosmology and Gravitation (ICG) for hospitality. This work was partially supported by NSFC under grant No. 10703005 and No. 10775119 (AW & QW).

References

[1] V.A. Rubakov, *Large and infinite extra dimensions: an introduction*, *Phys. Usp.* **44** (2001) 871 [*Usp. Fiz. Nauk* **171** (2001) 913] [hep-ph/0104152] [SPIRES];
R. Maartens, *Brane-world gravity*, *Living Rev. Rel.* **7** (2004) 7 [gr-qc/0312059] [SPIRES];
Dark energy from brane-world gravity, *J. Phys. Conf. Ser.* **68** (2007) 012046 [*Lect. Notes Phys.* **720** (2007) 323] [astro-ph/0602415] [SPIRES];
P. Brax, C. van de Bruck and A.C. Davis, *Brane world cosmology*, *Rept. Prog. Phys.* **67** (2004) 2183 [hep-th/0404011] [SPIRES];
C. Csaki, *TASI lectures on extra dimensions and branes*, hep-ph/0404096 [SPIRES];
V. Sahni, *Cosmological surprises from braneworld models of dark energy*, astro-ph/0502032 [SPIRES];
D. Langlois, *Is our Universe brany?*, *Prog. Theor. Phys. Suppl.* **163** (2006) 258 [hep-th/0509231] [SPIRES];
R. Durrer, *Braneworlds*, *AIP Conf. Proc.* **782** (2005) 202 [hep-th/0507006] [SPIRES];
A. Lue, *The phenomenology of Dvali-Gabadadze-Porrati cosmologies*, *Phys. Rept.* **423** (2006) 1 [astro-ph/0510068] [SPIRES];

D. Wands, *Brane-world cosmology*, *AIP Conf. Proc.* **841** (2006) 228 [gr-qc/0601078] [SPIRES].

[2] P. Hořava and E. Witten, *Heterotic and type I string dynamics from eleven dimensions*, *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209] [SPIRES]; *Eleven-dimensional supergravity on a manifold with boundary*, *Nucl. Phys.* **B 475** (1996) 94 [hep-th/9603142] [SPIRES].

[3] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *The universe as a domain wall*, *Phys. Rev.* **D 59** (1999) 086001 [hep-th/9803235] [SPIRES]; *Heterotic M-theory in five dimensions*, *Nucl. Phys.* **B 552** (1999) 246 [hep-th/9806051] [SPIRES].

[4] J.-L. Lehners, P. Smyth and K.S. Stelle, *Stability of Hořava-Witten spacetimes*, *Class. Quant. Grav.* **22** (2005) 2589 [hep-th/0501212] [SPIRES].

[5] Q. Wu, Y.G. Gong and A. Wang, *Brany cosmology in the Hořava-Witten heterotic M-theory on S^1/Z_2*, *JCAP* **06** (2009) 015 [arXiv:0810.5377] [SPIRES].

[6] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315] [SPIRES]; *Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity*, *Phys. Rev.* **D 59** (1999) 086004 [hep-ph/9807344] [SPIRES]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398] [SPIRES].

[7] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221] [SPIRES].

[8] J.M. Cline, C. Grojean and G. Servant, *Cosmological expansion in the presence of extra dimensions*, *Phys. Rev. Lett.* **83** (1999) 4245 [hep-ph/9906523] [SPIRES]; C. Csáki, M. Graesser, C.F. Kolda and J. Terning, *Cosmology of one extra dimension with localized gravity*, *Phys. Lett.* **B 462** (1999) 34 [hep-ph/9906513] [SPIRES].

[9] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Phenomenology of the Randall-Sundrum gauge hierarchy model*, *Phys. Rev. Lett.* **84** (2000) 2080 [hep-ph/9909255] [SPIRES]; G.F. Giudice, R. Rattazzi and J.D. Wells, *Graviscalars from higher-dimensional metrics and curvature-Higgs mixing*, *Nucl. Phys.* **B 595** (2001) 250 [hep-ph/0002178] [SPIRES]; G.D. Krib, *Physics of the radion in the Randall-Sundrum scenario*, *eConf* **C010630** (2001) P317 [hep-th/0110242] [SPIRES]; H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Experimental probes of localized gravity: on and off the wall*, *Phys. Rev.* **D 63** (2001) 075004 [hep-ph/0006041] [SPIRES]; T.G. Rizzo, *Radion couplings to bulk fields in the Randall-Sundrum model*, *JHEP* **06** (2002) 056 [hep-ph/0205242] [SPIRES]; D. Dominici, B. Grzadkowski, J.F. Gunion and M. Toharia, *The scalar sector of the Randall-Sundrum model*, *Nucl. Phys.* **B 671** (2003) 243 [hep-ph/0206192] [SPIRES]; J.F. Gunion, M. Toharia and J.D. Wells, *Precision electroweak data and the mixed radion-Higgs sector of warped extra dimensions*, *Phys. Lett.* **B 585** (2004) 295 [hep-ph/0311219] [SPIRES]; M.S. Carena, E. Ponton, J. Santiago and C.E.M. Wagner, *Electroweak constraints on warped models with custodial symmetry*, *Phys. Rev.* **D 76** (2007) 035006 [hep-ph/0701055] [SPIRES]; A.L. Fitzpatrick, J. Kaplan, L. Randall and L.-T. Wang, *Searching for the Kaluza-Klein graviton in bulk RS models*, *JHEP* **09** (2007) 013 [hep-ph/0701150] [SPIRES];

C. Csáki, J. Hubisz and S.J. Lee, *Radion phenomenology in realistic warped space models*, *Phys. Rev.* **D 76** (2007) 125015 [arXiv:0705.3844] [SPIRES];
H. Davoudiasl, T.G. Rizzo and A. Soni, *On direct verification of warped hierarchy-and-flavor models*, *Phys. Rev.* **D 77** (2008) 036001 [arXiv:0710.2078] [SPIRES];
O. Antipin and A. Soni, *Towards establishing the spin of warped gravitons*, *JHEP* **10** (2008) 018 [arXiv:0806.3427] [SPIRES];
L. Randall and M.B. Wise, *Exotic implications of electron and photon final states*, arXiv:0807.1746 [SPIRES] and references therein.

[10] S. Weinberg, *The cosmological constant problem*, *Rev. Mod. Phys.* **61** (1989) 1 [SPIRES];
S.M. Carroll, *The cosmological constant*, *Living Rev. Rel.* **4** (2001) 1 [astro-ph/0004075] [SPIRES];
T. Padmanabhan, *Cosmological constant: the weight of the vacuum*, *Phys. Rept.* **380** (2003) 235 [hep-th/0212290] [SPIRES];
S. Nobbenhuis, *Categorizing different approaches to the cosmological constant problem*, *Found. Phys.* **36** (2006) 613 [gr-qc/0411093] [SPIRES];
J. Polchinski, *The cosmological constant and the string landscape*, hep-th/0603249 [SPIRES];
J.M. Cline, *String cosmology*, hep-th/0612129 [SPIRES].

[11] C. Csaki, J. Erlich and C. Grojean, *Essay on gravitation: the cosmological constant problem in brane-worlds and gravitational Lorentz violations*, *Gen. Rel. Grav.* **33** (2001) 1921 [gr-qc/0105114] [SPIRES].

[12] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, *A small cosmological constant from a large extra dimension*, *Phys. Lett.* **B 480** (2000) 193 [hep-th/0001197] [SPIRES];
S. Kachru, M.B. Schulz and E. Silverstein, *Self-tuning flat domain walls in* 5*D gravity and string theory*, *Phys. Rev.* **D 62** (2000) 045021 [hep-th/0001206] [SPIRES].

[13] Y. Aghababaie, C.P. Burgess, S.L. Parameswaran and F. Quevedo, *Towards a naturally small cosmological constant from branes in* 6*D supergravity*, *Nucl. Phys.* **B 680** (2004) 389 [hep-th/0304256] [SPIRES]; *Warped brane worlds in six dimensional supergravity*, *JHEP* **09** (2003) 037 [hep-th/0308064] [SPIRES];
C.P. Burgess, *Supersymmetric large extra dimensions and the cosmological constant: an update*, *Ann. Phys.* **313** (2004) 283 [hep-th/0402200] [SPIRES]; *Towards a natural theory of dark energy: supersymmetric large extra dimensions*, *AIP Conf. Proc.* **743** (2005) 417 [hep-th/0411140] [SPIRES];
C.P. Burgess, J. Matias and F. Quevedo, *MSLED: a Minimal Supersymmetric Large Extra Dimensions scenario*, *Nucl. Phys.* **B 706** (2005) 71 [hep-ph/0404135] [SPIRES].

[14] S. Förste, Z. Lalak, S. Lavignac and H.P. Nilles, *A comment on self-tuning and vanishing cosmological constant in the brane world*, *Phys. Lett.* **B 481** (2000) 360 [hep-th/0002164] [SPIRES]; *The cosmological constant problem from a brane-world perspective*, *JHEP* **09** (2000) 034 [hep-th/0006139] [SPIRES];
C. Csáki, J. Erlich and C. Grojean, *Gravitational Lorentz violations and adjustment of the cosmological constant in asymmetrically warped spacetimes*, *Nucl. Phys.* **B 604** (2001) 312 [hep-th/0012143] [SPIRES];
J.M. Cline and H. Firouzjahi, *No-go theorem for horizon-shielded self-tuning singularities*, *Phys. Rev.* **D 65** (2002) 043501 [hep-th/0107198] [SPIRES].

[15] K. Koyama, *The cosmological constant and dark energy in braneworlds*, *Gen. Rel. Grav.* **40** (2008) 421 [arXiv:0706.1557] [SPIRES];

C.P. Burgess, *Extra dimensions and the cosmological constant problem*, arXiv:0708.0911 [SPIRES].

[16] G.R. Dvali, G. Gabadadze and M. Porrati, *Metastable gravitons and infinite volume extra dimensions*, *Phys. Lett.* **B 484** (2000) 112 [hep-th/0002190] [SPIRES].

[17] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064] [SPIRES].

[18] C. Deffayet, *Cosmology on a brane in Minkowski bulk*, *Phys. Lett.* **B 502** (2001) 199 [hep-th/0010186] [SPIRES];
V. Sahni and Y. Shtanov, *Braneworld models of dark energy*, *JCAP* **11** (2003) 014 [astro-ph/0202346] [SPIRES];
A. Lue, *The phenomenology of Dvali-Gabadadze-Porrati cosmologies*, *Phys. Rept.* **423** (2006) 1 [astro-ph/0510068] [SPIRES];
R. Maartens, *Dark energy from brane-world gravity*, *J. Phys. Conf. Ser.* **68** (2007) 012046 [*Lect. Notes Phys.* **720** (2007) 323] [astro-ph/0602415] [SPIRES] and references therein.

[19] M.A. Luty, M. Porrati and R. Rattazzi, *Strong interactions and stability in the DGP model*, *JHEP* **09** (2003) 029 [hep-th/0303116] [SPIRES];
A. Nicolis and R. Rattazzi, *Classical and quantum consistency of the DGP model*, *JHEP* **06** (2004) 059 [hep-th/0404159] [SPIRES];
K. Koyama, *Are there ghosts in the self-accelerating brane universe?*, *Phys. Rev.* **D 72** (2005) 123511 [hep-th/0503191] [SPIRES];
D. Gorbunov, K. Koyama and S. Sibiryakov, *More on ghosts in DGP model*, *Phys. Rev.* **D 73** (2006) 044016 [hep-th/0512097] [SPIRES];
C. Charmousis, R. Gregory, N. Kaloper and A. Padilla, *DGP specteroscopy*, *JHEP* **10** (2006) 066 [hep-th/0604086] [SPIRES];
R. Gregory, N. Kaloper, R.C. Myers and A. Padilla, *A new perspective on DGP gravity*, *JHEP* **10** (2007) 069 [arXiv:0707.2666] [SPIRES];
K. Koyama, *Ghosts in the self-accelerating universe*, *Class. Quant. Grav.* **24** (2007) R231 [arXiv:0709.2399] [SPIRES].

[20] C. Deffayet, G. Gabadadze and A. Iglesias, *Perturbations of self-accelerated universe*, *JCAP* **08** (2006) 012 [hep-th/0607099] [SPIRES];
G. Dvali, *Predictive power of strong coupling in theories with large distance modified gravity*, *New J. Phys.* **8** (2006) 326 [hep-th/0610013] [SPIRES].

[21] W. Fang et al., *Challenges to the DGP model from horizon-scale growth and geometry*, *Phys. Rev.* **D 78** (2008) 103509 [arXiv:0808.2208] [SPIRES].

[22] Y. Gong, M. Ishak and A. Wang, *Growth factor parametrization in curved space*, *Phys. Rev.* **D 80** (2009) 023002 [arXiv:0903.0001] [SPIRES].

[23] K. Benakli, *Cosmological solution in M-theory on S^1/Z_2*, *Int. J. Mod. Phys.* **D 8** (1999) 153 [hep-th/9804096] [SPIRES];
H.S. Reall, *Open and closed cosmological solutions of Hořava-Witten theory*, *Phys. Rev.* **D 59** (1999) 103506 [hep-th/9809195] [SPIRES];
A. Lukas, B.A. Ovrut and D. Waldram, *Cosmological solutions of Hořava-Witten theory*, *Phys. Rev.* **D 60** (1999) 086001 [hep-th/9806022] [SPIRES].

[24] W. Chen, Z.W. Chong, G.W. Gibbons, H. Lu and C.N. Pope, *Hořava-Witten stability: eppur si muove*, *Nucl. Phys.* **B 732** (2006) 118 [hep-th/0502077] [SPIRES];
J.-L. Lehners, P. McFadden and N. Turok, *Colliding branes in heterotic M-theory*, *Phys. Rev.* **D 75** (2007) 103510 [hep-th/0611259] [SPIRES].

[25] Y. Gong, A. Wang and Q. Wu, *Cosmological constant and late transient acceleration of the universe in the Hořava-Witten Heterotic M-theory on S^1/Z_2*, *Phys. Lett.* **B 663** (2008) 147 [arXiv:0711.1597] [SPIRES].

[26] A. Wang and N.O. Santos, *The cosmological constant in the brane world of string theory on S^1/Z_2*, *Phys. Lett.* **B 669** (2008) 127 [arXiv:0712.3938] [SPIRES].

[27] Q. Wu, N.O. Santos, P. Vo and A. Wang, *Late transient acceleration of the universe in string theory on S^1/Z_2*, *JCAP* **09** (2008) 004 [arXiv:0804.0620] [SPIRES].

[28] A. Wang and N.O. Santos, *The hierarchy problem, the 4D Newtonian potential and its Yukawa corrections in string theory on S^1/Z_2*, *Phys. Lett.* **B 669** (2008) 127 [arXiv:0808.2055] [SPIRES].

[29] W. Fischler, A. Kashani-Poor, R. McNees and S. Paban, *The acceleration of the universe, a challenge for string theory*, *JHEP* **07** (2001) 003 [hep-th/0104181] [SPIRES];
J.M. Cline, *Quintessence, cosmological horizons and self-tuning*, *JHEP* **08** (2001) 035 [hep-ph/0105251] [SPIRES];
E. Halyo, *Hybrid quintessence with an end or quintessence from branes and large dimensions*, *JHEP* **10** (2001) 025 [hep-ph/0105216] [SPIRES];
S. Hellerman, N. Kaloper and L. Susskind, *String theory and quintessence*, *JHEP* **06** (2001) 003 [hep-th/0104180] [SPIRES].

[30] L.M. Krauss and R.J. Scherrer, *The return of a static universe and the end of cosmology*, *Gen. Rel. Grav.* **39** (2007) 1545 [*Int. J. Mod. Phys.* **D 17** (2008) 685] [arXiv:0704.0221] [SPIRES] and references therein.

[31] J.E. Lidsey, D. Wands and E.J. Copeland, *Superstring cosmology*, *Phys. Rept.* **337** (2000) 343 [hep-th/9909061] [SPIRES].

[32] T. Battefeld and S. Watson, *String gas cosmology*, *Rev. Mod. Phys.* **78** (2006) 435 [hep-th/0510022] [SPIRES].

[33] A. Tziolas, A. Wang and Z.C. Wu, *Colliding branes and formation of spacetime singularities in string theory*, *JHEP* **04** (2009) 038 [arXiv:0812.1377] [SPIRES];
P. Sharma, A. Tziolas and A. Wang, *Spacetime singularities in string and its low dimensional effective theory*, arXiv:0901.2676 [SPIRES].

[34] P.K. Townsend and M.N.R. Wohlfarth, *Accelerating cosmologies from compactification*, *Phys. Rev. Lett.* **91** (2003) 061302 [hep-th/0303097] [SPIRES].

[35] G.W. Gibbons, *Aspects of supergravity theories*, in *Supersymmetry, Supergravity and Related Topics*, F. de Aguila et al. eds., World Scientific, Singapore (1985), pg. 124;
J.M. Maldacena and C. Núñez, *Supergravity description of field theories on curved manifolds and a no go theorem*, *Int. J. Mod. Phys.* **A 16** (2001) 822 [hep-th/0007018] [SPIRES].

[36] N. Ohta, *Accelerating cosmologies from S-branes*, *Phys. Rev. Lett.* **91** (2003) 061303 [hep-th/0303238] [SPIRES].

[37] M.N.R. Wohlfarth, *Accelerating cosmologies and a phase transition in M-theory*, *Phys. Lett.* **B 563** (2003) 1 [hep-th/0304089] [SPIRES];
S. Roy, *Accelerating cosmologies from M/string theory compactifications*, *Phys. Lett.* **B 567** (2003) 322 [hep-th/0304084] [SPIRES].

[38] J.K. Webb et al., *Further evidence for cosmological evolution of the fine structure constant*, *Phys. Rev. Lett.* **87** (2001) 091301 [astro-ph/0012539] [SPIRES];

JHEP10(2009)095

J.M. Cline and J. Vinet, *Problems with time-varying extra dimensions or 'Cardassian expansion' as alternatives to dark energy*, *Phys. Rev.* **D 68** (2003) 025015 [hep-ph/0211284] [SPIRES];
N. Ohta, *A study of accelerating cosmologies from superstring/M theories*, *Prog. Theor. Phys.* **110** (2003) 269 [hep-th/0304172] [SPIRES];
N. Ohta, *Accelerating cosmologies and inflation from M/superstring theories*, *Int. J. Mod. Phys.* **A 20** (2005) 1 [hep-th/0411230] [SPIRES];
C.-M. Chen, P.-M. Ho, I.P. Neupane, N. Ohta and J.E. Wang, *Hyperbolic space cosmologies*, *JHEP* **10** (2003) 058 [hep-th/0306291] [SPIRES];
E. Bergshoeff, A. Collinucci, U. Gran, M. Nielsen and D. Roest, *Transient quintessence from group manifold reductions or how all roads lead to Rome*, *Class. Quant. Grav.* **21** (2004) 1947 [hep-th/0312102] [SPIRES];
Y. Gong and A. Wang, *Acceleration from M-theory and fine-tuning*, *Class. Quant. Grav.* **23** (2006) 3419 [gr-qc/0602055] [SPIRES];
I.P. Neupane and D.L. Wiltshire, *Cosmic acceleration from M theory on twisted spaces*, *Phys. Rev.* **D 72** (2005) 083509 [hep-th/0504135] [SPIRES]; *Accelerating cosmologies from compactification with a twist*, *Phys. Lett.* **B 619** (2005) 201 [hep-th/0502003] [SPIRES];
K.-I. Maeda and N. Ohta, *Inflation from M-theory with fourth-order corrections and large extra dimensions*, *Phys. Lett.* **B 597** (2004) 400 [hep-th/0405205] [SPIRES]; *Inflation from superstring/M theory compactification with higher order corrections. I*, *Phys. Rev.* **D 71** (2005) 063520 [hep-th/0411093] [SPIRES];
K. Vandersloot, *On the Hamiltonian constraint of loop quantum cosmology*, *Phys. Rev.* **D 71** (2005) 103506 [gr-qc/0502082] [SPIRES];
V. Baukh and A. Zhuk, *Sp-brane accelerating cosmologies*, *Phys. Rev.* **D 73** (2006) 104016 [hep-th/0601205] [SPIRES];
A. Krause, *Supersymmetry breaking with zero vacuum energy in M-theory flux compactifications*, *Phys. Rev. Lett.* **98** (2007) 241601 [hep-th/0701009] [SPIRES];
I.P. Neupane, *Accelerating universes from compactification on a warped conifold*, *Phys. Rev. Lett.* **98** (2007) 061301 [hep-th/0609086] [SPIRES] and references therein.

[39] R. Altendorfer, J. Bagger and D. Nemeschansky, *Supersymmetric Randall-Sundrum scenario*, *Phys. Rev.* **D 63** (2001) 125025 [hep-th/0003117] [SPIRES];
A. Falkowski, Z. Lalak and S. Pokorski, *Supersymmetrizing branes with bulk in five-dimensional supergravity*, *Phys. Lett.* **B 491** (2000) 172 [hep-th/0004093] [SPIRES]; *Five-dimensional gauged supergravities with universal hypermultiplet and warped brane worlds*, *Phys. Lett.* **B 509** (2001) 337 [hep-th/0009167] [SPIRES];
J. Bagger and D.V. Belyaev, *Supersymmetric branes with (almost) arbitrary tensions*, *Phys. Rev.* **D 67** (2003) 025004 [hep-th/0206024] [SPIRES].

[40] E. Bergshoeff, R. Kallosh and A. Van Proeyen, *Supersymmetry in singular spaces*, *JHEP* **10** (2000) 033 [hep-th/0007044] [SPIRES];
M. Zucker, *Supersymmetric brane world scenarios from off-shell supergravity*, *Phys. Rev.* **D 64** (2001) 024024 [hep-th/0009083] [SPIRES].

[41] M. Cvetič, H. Lü and C.N. Pope, *Domain walls and massive gauged supergravity potentials*, *Class. Quant. Grav.* **17** (2000) 4867 [hep-th/0001002] [SPIRES].

[42] M. Gasperini, *Elements of string cosmology*, Cambridge University Press, Cambridge U.K. (2007) [SPIRES].

[43] F. Leblond, R.C. Myers and D.J. Winters, *Consistency conditions for brane worlds in arbitrary dimensions*, *JHEP* **07** (2001) 031 [hep-th/0106140] [SPIRES].

[44] P. Binétruy, C. Deffayet, U. Ellwanger and D. Langlois, *Brane cosmological evolution in a bulk with cosmological constant*, *Phys. Lett.* **B 477** (2000) 285 [hep-th/9910219] [SPIRES];
P. Binétruy, C. Deffayet and D. Langlois, *The radion in brane cosmology*, *Nucl. Phys.* **B 615** (2001) 219 [hep-th/0101234] [SPIRES].

[45] A. Wang, R.-G. Cai and N.O. Santos, *Accelerating universe in Randall-Sundrum models of two* 3*-branes*, *Nucl. Phys.* **B 797** (2008) 395 [astro-ph/0607371] [SPIRES].

[46] T. Shiromizu, K.-I. Maeda and M. Sasaki, *The Einstein equations on the* 3*-brane world*, *Phys. Rev.* **D 62** (2000) 024012 [gr-qc/9910076] [SPIRES];
A.N. Aliev and A.E. Gumrukcuoglu, *Gravitational field equations on and off a* 3*-brane world*, *Class. Quant. Grav.* **21** (2004) 5081 [hep-th/0407095] [SPIRES];
R.-G. Cai and L.-M. Cao, *Thermodynamics of apparent horizon in brane world scenario*, *Nucl. Phys.* **B 785** (2007) 135 [hep-th/0612144] [SPIRES].

[47] C. Lanczos, *Bemerkung zur de Sitterschen Welt* (in German), *Phys. Z.* **23** (1922) 539; *Flächenhafte Verteilung der Materie in der Einsteinschen Gravitationstheorie* (in German), *Annalen Phys.* **379** (1924) 518.

[48] J. Martin, G.N. Felder, A.V. Frolov, M. Peloso and L. Kofman, *Braneworld dynamics with the BraneCode*, *Phys. Rev.* **D 69** (2004) 084017 [hep-th/0309001] [SPIRES].

[49] W.D. Goldberger and M.B. Wise, *Modulus stabilization with bulk fields*, *Phys. Rev. Lett.* **83** (1999) 4922 [hep-ph/9907447] [SPIRES]; *Phenomenology of a stabilized modulus*, *Phys. Lett.* **B 475** (2000) 275 [hep-ph/9911457] [SPIRES].

[50] M. Abramowitz and I.A. Stegun, *Handbook of mathematical functions*, Dover Publications Inc., New York U.S.A. (1972), pg. 374.

[51] http://dx.doi.org/10.1103/PhysRevLett.84.2778 J. Garriga and T. Tanaka, *Gravity in the brane-world*, *Phys. Rev. Lett.* **84** (2000) 2778 [hep-th/9911055] [SPIRES];
T. Tanaka and X. Montes, *Gravity in the brane-world for two-branes model with stabilized modulus*, *Nucl. Phys.* **B 582** (2000) 259 [hep-th/0001092] [SPIRES].

[52] C. Csáki, J. Erlich, T.J. Hollowood and Y. Shirman, *Universal aspects of gravity localized on thick branes*, *Nucl. Phys.* **B 581** (2000) 309 [hep-th/0001033] [SPIRES].

[53] C. Csáki, M.L. Graesser and G.D. Kribs, *Radion dynamics and electroweak physics*, *Phys. Rev.* **D 63** (2001) 065002 [hep-th/0008151] [SPIRES].

[54] C. Cohen-Tannoudji, B. Diu and F. Laloë, *Quantum mechanics*, volume I, John Wiley & Sons, New York U.S.A. (1977); *Quantum mechanics*, volume II, John Wiley & Sons, New York U.S.A. (1977).

[55] A. Brandhuber and K. Sfetsos, *Non-standard compactifications with mass gaps and Newton's law*, *JHEP* **10** (1999) 013 [hep-th/9908116] [SPIRES].

[56] V. Sahni and A.A. Starobinsky, *The case for a positive cosmological* Λ*-term*, *Int. J. Mod. Phys.* **D 9** (2000) 373 [astro-ph/9904398] [SPIRES];
P.J.E. Peebles and B. Ratra, *The cosmological constant and dark energy*, *Rev. Mod. Phys.* **75** (2003) 559 [astro-ph/0207347] [SPIRES];
T. Padmanabhan, *Cosmological constant: the weight of the vacuum*, *Phys. Rept.* **380** (2003) 235 [hep-th/0212290] [SPIRES];

JHEP10(2009)095

V. Sahni, *Dark matter and dark energy*, *Lect. Notes Phys.* **653** (2004) 141 [astro-ph/0403324] [SPIRES];
E.J. Copeland, M. Sami and S. Tsujikawa, *Dynamics of dark energy*, *Int. J. Mod. Phys.* **D 15** (2006) 1753 [hep-th/0603057] [SPIRES];
E.V. Linder, *Mapping the cosmological expansion*, *Rept. Prog. Phys.* **71** (2008) 056901 [arXiv:0801.2968] [SPIRES];
J. Frieman, M. Turner and D. Huterer, *Dark energy and the accelerating universe*, *Ann. Rev. Astron. Astrophys.* **46** (2008) 385 [arXiv:0803.0982] [SPIRES].